Returning a Treasure: Revisiting the Asymmetric Matching Pennies Contradiction

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Abstract[1]:

The asymmetric matching pennies contradiction posed by Goeree and Holt (2001; "Ten Little Treasures of Game Theory and Ten Intuitive Contradictions") posits that contrary to the prediction of mixed strategy Nash equilibrium, experimental subjects' choices are in practice, based heavily on the magnitudes of their *own payoffs*. Own-payoff effects are robustly confirmed in several separate studies in the literature (Ochs, 1995; McKelvey, Palfrey and Weber, 2000; Goeree, Holt and Palfrey, 2003). However, recent psychology literature finds that people from collectivist cultures are substantially more adept at taking the perspective of others compared to people from individualist cultures, suggesting that such populations may more readily apply the reasoning needed to obtain mixed strategy equilibrium. Closely following the experimental setups of Goeree and Holt (2001) and related studies, we conduct a series of asymmetric matching pennies games in China, hypothesizing play which is closer to equilibrium frequencies. Contrary to previous experiments which were conducted in the United States, we find that the behavior of our subjects is in fact very close to the mixed strategy equilibrium, and there are essentially no own-payoff effects among Row players who face the large payoff asymmetry. In a Quantal Response Equilibrium framework allowing for altruism or spite, the behavior of our subjects corresponded to a positive spite parameter, whereas the results of previous studies corresponded to altruism. Our results highlight the importance of testing behavior in games across populations and cultures, before generalizing behavioral anomalies to humans as a whole (Henrich, Heine and Norenzayan, 2010; Henrich, Boyd, Bowles, Camerer, Fehr and Gintis, 2004).

Keywords: mixed strategy equilibria, experiments, asymmetric matching pennies, cultural difference
JEL codes: C70, C72, C91

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1. Introduction

Mixed strategy equilibria are critical for the existence of Nash equilibrium in games, but have posed both theoretical and empirical challenges in justifying their use as a solution concept. The classic theoretical concerns address how to justify players' pure strategy mixtures in equilibrium, when each player mixes so as to make the other player fully indifferent between any mixing over non-dominated pure strategies (see Osborne and Rubinstein, 2001 for a discussion). However, in addition to the theoretical issues, there are also challenges in terms of empirical validity.

Goeree and Holt (2001; "Ten Little Treasures of Game Theory and Ten Intuitive Contradictions"), found that for the case of symmetric matching pennies games with unique mixed strategy equilibrium, the equilibrium prediction holds up fairly well on human subjects. However, in the case of a highly asymmetric payoff version of the game, the equilibrium prediction fails gravely as players seem to gravitate heavily towards their own highest payoff opportunity, rather than randomizing such that their opponent has no room to exploit them in expectation. Their hypothesis is that the concept of mixed strategy equilibrium seems to only work coincidentally in the case of symmetry, a conjecture which if shown to be true, implies the mixed strategy equilibrium concept may not be an appropriate description of human strategic behavior.

Other studies have robustly confirmed that subjects display "own-payoff" effects in the laboratory for the asymmetric matching pennies class of games, under a variety of conditions. Ochs (1995) studies asymmetric matching pennies games under repetition, finding that even after 50 rounds of repetition, subjects still gravitate significantly more than equilibrium predicts, toward their own highest payoff. These effects are confirmed in the experimental results of McKelvey, Palfrey and Weber (2000), and Goeree, Holt and Palfrey (2003).

In this paper, we revisit the asymmetric matching pennies contradiction, under the hypothesis that subjects in China might restore empirical validity to the mixed strategy Nash equilibrium concept, even under high levels of payoff asymmetry. A recent paper by Henrich, Heine, and Norenzayan (2010) raises the concern about drawing generalized conclusions about human psychology and behavior from experiments on "Western, Educated, Industrialized, Rich and Democratic (WEIRD) societies". Their general concern is well-reflected in several experiments which were conducted in 'small-scale' societies in Henrich, Boyd, Bowles, Camerer, Fehr and Gintis (2004). Even more specifically to our study, in research on perspective-taking among individuals from collectivist versus individualist societies, Wu and Keysar (2007) and Wu, Barr, Gann and Keysar (2013) find that subjects from China performed better overall (and self-corrected more rapidly) at tasks in which they needed to consider the viewpoint of another person. In our asymmetric matching pennies games, considering another person's perspective, corresponds to paying attention to the other player's payoffs, which is exactly what is needed for subjects to reach equilibrium strategies. Subjects cannot infer the mixed strategy equilibrium in these games from focusing on their own payoff possibilities.

A related explanatory factor is prior exposure to game theory and strategic concepts. Our subjects reported a relatively high rate of personal exposure (either formal or informal) to game theory. Some experiments (notably McKelvey, Palfrey and Weber (2000) in this literature) specifically recruit subjects with limited or no prior training in game theory. The benefits of this approach are clear in that it avoids the situation where the experiment merely tests subjects' prior learned knowledge. However, there can also be drawbacks to this screening approach, particularly if the population that subjects are drawn from is generally inclined to be interested in strategic and game theoretic issues to begin with. In particular, the study of strategic interaction has been a traditional Chinese intellectual activity for thousands of years.[2] We briefly discuss the appeal of game theory in Chinese society in

[2] For example, the famous 2500-year-old classical Chinese text 孙子兵法 (The Art of War, by Sun WU).

the conclusion, and leave the further exploration of this connection to future work.

Our subjects play repeatedly under either random re-matching or fixed partner matching, and can observe their history of play. We find that our subjects adhered to the equilibrium prediction in the aggregate much more closely compared to other studies. Specifically, our Row players, whose payoffs are highly asymmetric, tend to play pure strategy frequencies very close to the equilibrium prediction as dictated by his or her partner's payoffs. They do not show any sign of the "own-payoff" effects prevalent in previous studies. Our Column players, whose equilibrium mixture is very asymmetric as dictated by Row player's payoffs, show evidence of learning dynamics. That is, over time, Column players' randomization in the aggregate, moves significantly in the direction predicted by equilibrium. The difference between our results and the previous results in the literature can also been understood in a Quantal Response Equilibrium framework which allows for altruism or spite. While the behavior of subjects in Goeree and Holt (2001) corresponds to players with altruistic preferences (Levine and Zheng, forthcoming), our players' behavior corresponds to spiteful preferences.

There is substantial individual level heterogeneity in terms of the tendency to randomize pure strategy play according to the mixed strategy equilibrium. Subjects in our experiment answered an exit questionnaire in which we asked questions to gauge their familiarity with game theory as well as their potential social preferences. We found that Row players' personal deviation from the equilibrium mixture was significantly lower if they had taken a game theory class before. Having previously taken a game theory class had no correlation with Column players' individual behavior.

Our paper contributes to the behavioral game theory literature which seeks to understand the discrepancies between theoretical mixed strategy Nash equilibria and their implementation by human subjects in the laboratory. Ochs (1995) finds violations of own-payoff invariance predicted by mixed strategy equilibria in a series of matching pennies games. McKelvey and Palfrey (1995) propose Quantal Response Equilibrium, or stochastic best response, as a better fit to the data. Goeree, Holt and Palfrey (2003) propose risk averse players in a Quantal Response Equilibrium setting in order to better fit the data. Martin, Bhui, Bossaerts, Matsuzawa and Camerer (2014) find that chimpanzees play closer to equilibrium in asymmetric matching pennies games than humans do, and suggest an evolutionary interpretation of game theory.

Other relevant papers have proposed different explanations for several of the Ten Intuitive Contradictions in Goeree and Holt (2001). Erlei (2008) proposes that social preferences can explain most of the ten contradictory findings, including the asymmetric matching pennies puzzle. Eichberger and Kelsey (2011) argue that ambiguity aversion over the other player's strategy choice can explain Goeree and Holt's contradictions for normal form games, including the matching pennies case.

Our study differs from these works not in terms of methodology or experimental design, but in terms of our result, which is very close to the equilibrium prediction. Our Row players did not show strong own-payoff tendencies as found in other papers. On the other hand, while our Column players did not play their equilibrium strategy as closely as found in Goeree and Holt (2001), they moved significantly towards equilibrium over time, suggestive of learning dynamics. In a Quantal Response Equilibrium framework with the possibility of other-regarding preferences, behavior in our experiments corresponded best to a specification with a positive spite parameter (Levine, 1998; Levine and Zheng, forthcoming). While the equilibrium prediction holds up relatively well in the aggregate, we also explore individual differences and possible explanatory factors. Randomizing close to one's equilibrium strategy is significantly predicted by previous formal training in game theory for Row players, although not for Column players.

Finally, our paper is related to a larger literature on behavior in games with mixed strategy equilibria originating with O'Neill (1987) and Brown and Rosenthal (1990). Shachat (2002) investigates subjects' failure to play the unique minimax mixed strategy equilibrium, and derives a method to distinguish between subjects' mixed strategy intentions and their actual implementations. Shachat, Swarthout and Wei (forthcoming) develop a statistical model to detect mixed strategy play in repeated games. Belot, Crawford and Heyes (2013) find that in matching pennies games, subjects who can observe their opponents gestures have a tendency to automatically imitate their opponents' action.

The remainder of the paper proceeds as follows: Section 2 describes our experimental setup; Section 3 provides our aggregate results; Section 4 explores time dynamics; Section 5 discusses individual heterogeneity; Section 6 accounts for the results using ε-equilbrium and Quantal Response Equilibrium concepts; Section 7 provides additional results from fixed-partner matching treatments, which yielded similar results; Section 8 concludes and discusses.

2. Experiment Description

We used the asymmetric matching pennies games proposed by Goeree and Holt (2001). Their "Asymmetric Matching Pennies" is identical to our "Large Asymmetry" game, and their "Reversed Asymmetry" is identical to our "Small Asymmetry" game, where 'reverse' in their terminology, refers to the pure strategy B having the highest possible payoff for Row player rather than T as in the original asymmetric games. Figure 1 shows the two games and their equilibrium strategy profiles, which as usual, are found by each player randomizing among his pure strategies so as to make the other player indifferent between either of his own pure strategies.

Figure 1: Matching Pennies Games

Large Asymmetry

equilibrium: (T:0.5, B: 0.5; L:0.125, R:0.875)

	L	R
T	(32, 4)	(4,8)
B	(4,8)	(8,4)

Small Asymmetry

equilibrium: (T:0.5, B: 0.5; L:0.909, R:0.091)

	L	R
T	(4.4, 4)	(4,8)
B	(4,8)	(8,4)

Payoffs expressed in Experimental Currency Units (ECU), which convert to RMB at the rate 10ECU = 1RMB.

Figure 2 shows the results from Goeree and Holt (2001), which were conducted on 50 subjects in 5 cohorts of 10, randomly matched for one-shot play. Payoffs in the table are expressed in cents. Their results show a striking violation of the equilibrium prediction. Compared to the Symmetric Matching Pennies case where subjects randomized in a manner very close to that predicted by the equilibrium (T: 0.5, B: 0.5; L: 0.5, R: 0.5), the other two games show that the row player seems to be *highly* driven towards the pure strategy which has the potential of yielding his highest payoff opportunity. The violation is stark in that 96% of the Row players chose Top in the Asymmetric case and 92% of subjects chose Bottom in the Reversed case. The Column players tended to correctly anticipate Row players' behavior, best-responding accordingly.

Our experimental sessions were conducted on June 10^{th}, 2012 in the Tsinghua University School of Economics and Management, Economic Science and Policy Experimental Laboratory (ESPEL). Each session lasted about one hour, and consisted of two treatments. Subjects were recruited using the ORSEE system. The experiments were conducted using z-Tree software (Fischbacher, 2007).

Table 1 shows the summary of each session in terms of treatment ordering, partner matching, subject earnings and number of subjects. In each session subjects were paired randomly each period to play the game. Each treatment within each session contained 50 rounds of Asymmetric Matching

Pennies, with a history table which listed subjects' personal history of play and winnings in each period. Subjects were randomly assigned their roles (Row or Column player) and maintained fixed roles within each treatment of their session. The game was described to subjects using sentences describing each player's payoff conditional on their choice and their partner's choice, rather than teaching subjects how to read a game matrix. The details of the experimental instructions can be found in the Appendix.

Figure 2: Goeree and Holt (2001) Results (Table 1)

TABLE 1—THREE ONE-SHOT MATCHING PENNIES GAMES (WITH CHOICE PERCENTAGES)

		Left (48)	*Right* (52)
Symmetric matching pennies	*Top* (**48**)	80, 40	40, 80
	Bottom (**52**)	40, 80	80, 40
		Left (16)	*Right* (84)
Asymmetric matching pennies	*Top* (**96**)	320, 40	40, 80
	Bottom (**4**)	40, 80	80, 40
		Left (80)	*Right* (20)
Reversed asymmetry	*Top* (**8**)	44, 40	40, 80
	Bottom (**92**)	40, 80	80, 40

Table 1: Session Summary

	Session 1	Session 2
Partner Matching	Random	Random
1st Treatment (Asymmetry)	Large	Small
2nd Treatment (Asymmetry)	Small	Large
Average earnings	75.4	75.3
Standard deviation	8.1	7.7
Minimum earnings	68	67.2
Maximum earnings	103.2	103.2
Number of subjects	42	42

3. Aggregate Results

Table 2 shows the frequency of pure strategies played in the aggregate, for each session, expressed for brevity in terms of the frequency of B choices by Row players and the frequency of R choices by Column players. The randomization as predicted by equilibrium is shown in the leftmost column.

Table 2: Aggregate Statistics

	Session 1	Session 2
Large Asymmetry (50% B; 87.5% R)	**(47% B; 72% R)**	(47% B; 75% R)
Small Asymmetry (50% B; 9% R)	(58% B; 29% R)	**(62% B; 27% R)**

Note: bold indicates first treatment played

Our results in both Sessions 1 and 2 are quite different from the results in Goeree and Holt (2001). Our Row players tended to play very close to the equilibrium prediction on average –

remarkably so in the case of the Large Asymmetry, such that the difference is indistinguishable from subjects' performance in the Symmetric matching pennies game in Goeree and Holt (2001). The Large Asymmetry result is consistent across sessions. In our Small Asymmetry treatments, Row players adhered slightly less to the equilibrium proportions than our Large Asymmetry results, but were quite far from the level of deviations seen in Goeree and Holt's Reversed asymmetry results.

Our results suggest that Row players in Asymmetric Matching Pennies games do not merely respond to their own most attractive payoff possibilities as suggested in Goeree and Holt (2001). Compared to their experiment, ours had subjects playing several times, while reminding them of their history of play in each round. These features of the experiment are likely to have assisted Row players in randomizing according to their equilibrium strategy. As we discuss in Section 4, for Row players the assisting feature seems to be the prospect of repetition, rather than learning dynamics.

Our Column players generally adhered slightly less well to the equilibrium prediction compared to Goeree and Holt's results, and in comparison to our own Row players. In the Large Asymmetry treatment, Column players tended to under-play R, while in the Small Asymmetry treatment, Column players tended to over-play R. However, as the next section shows, Column players tended to converge towards equilibrium frequencies over time.

4. Time Dynamics

So far we have only discussed the aggregate proportions of pure strategies played in the treatments. An additional question is whether subjects' initial responses adhered to the equilibrium frequencies.

If initial responses adhered well to equilibrium frequencies, it suggests that it is not the implementation of repetition which induces Row players to equilibrium, but the prospect of playing the game several times which encourages players to play in a randomized manner. One possibility in the one shot game is that players know that they should be indifferent between their two pure strategies; yet when actually implementing that randomization, they prefer the action which has the possibility of yielding the high payoff as a tie-breaker.

Table 3 shows the initial responses of subjects in the first two rounds of play, where we arbitrarily look at the first two rounds rather than just the first round in order to allow for some within-person averaging. The frequencies show that even considering just the first two rounds of play, Row players were approximately playing equilibrium. Column players were in the case of the 2nd treatment of the session (non-bold), closer to equilibrium proportions than in the aggregate over all rounds, but were equal or worse in the cases of the 1st treatments of the session (bold).

Table 3: Initial Responses (Pure strategy frequencies in first two rounds)

	Session 1	Session 2
Large Asymmetry (50% B; 87.5% R)	**(50% B; 71% R)**	(52% B; 90% R)
Small Asymmetry (50% B; 9% R)	(52% B; 26% R)	**(60% B; 40% R)**

Note: bold indicates first treatment played

The initial responses suggest that while playing many rounds is not necessary for frequencies of actions close to the equilibrium prediction, subjects' knowledge of the fact that they will play several rounds may help in realizing equilibrium frequencies. However, one question is whether over time, the frequencies get even closer to equilibrium.

We note that in our aggregate results, Row players are already playing very close to equilibrium frequencies. Column players have room to improve over time. We estimate a simple linear regression to detect the time trend in subjects' play, with proportion of the specified pure strategy played at time t by subject i as the dependent variable. A constant and a linear time trend are the explanatory variables.

$$proportion_{i,t} = \alpha + \beta \cdot t + \varepsilon_{i,t}$$

As in the previous section, we are primarily interested in picking up aggregate trends in the pure strategies played. Thus we estimate only aggregate coefficients while assuming simplistically that the errors are uncorrelated. We address individual heterogeneity in Section 5.

As Table 4 shows, in three out of the four treatments, Column players' adjustment of pure strategies over time was significant in the correct direction towards the equilibrium mixed strategy. For example, in the Session 1 Large Asymmetry treatment, Column players played R 71% of the time, whereas equilibrium would suggest they play R 87.5% of the time. The regression shows that column players played R about 66% of the time as a baseline, and increased their percentage of R play 0.25% points in each round (or an increase of 1% point every 4 rounds) on average. Similarly in the Small Asymmetry game, Column players were expected to play R 9% of the time, yet on the whole played R about 3 times as often as this equilibrium prediction. However as the time trend coefficients show, Column players adjusted their play of R significantly downward over time. Row players tended to show no particular time adjustment in their decisions.

The exception to this pattern is the Large asymmetry game in Session 2, where Row players adjusted their strategies in the correct direction, but Column players showed no significant adjustment towards equilibrium.

Table 4: Time Trend Regressions

	Session 1		Session 2	
Large asymmetry	*row player*	*column player*	*row player*	*column player*
Empirical frequency	**51% B**	**71% R**	47% B	75% R
Constant	0.4500**	0.6588**	0.5448**	0.7525**
Time trend coefficient	0.0007	0.0025**	-0.0029**	-0.0003
Small asymmetry	*row player*	*column player*	*row player*	*column player*
Empirical frequency	57% B	29% R	**62% B**	**27% R**
Constant	0.5745**	0.3504**	0.6083**	0.3572**
Time trend coefficient	0.0000	-0.0025**	0.0004	-0.0033**

*Note: bold indicates first treatment played ; ** denotes significance at 95% level*

5. Individual Analysis

While our results so far have been presented in the aggregate, one question is how well individual subjects adhere to the equilibrium. Our experimental results seem to suggest that most of the ability to fit the equilibrium prediction is from the cross-sectional dimension, rather than within subjects. We are interested in the individual proportions of B and R played, for the Large Asymmetry game and the Small Asymmetry game.

Each panel in Figure 3 shows proportion of the specified pure strategy (B for Row player, R for Column player) on the x-axis, and number of subjects on the y-axis. The equilibrium strategy proportion is denoted with the red vertical line. As the top row of panels show, there was wide individual variance in the proportion of B played by Row players across the entire 50 rounds. There is substantial amount of heterogeneity, even though when averaged across individuals, the proportion is close to equilibrium. The bottom row of panels shows the analogous charts for Column players play of R. The results are supportive of equilibrium in that almost all individuals were randomizing in the correct half of the possible range; yet most subjects were not implementing equilibrium on an individual basis.

Figure 4 shows essentially the same story for the Small Asymmetry game case. Note that Column players' distribution of personal randomization is skewed in the correct direction towards the equilibrium prediction, in both Figure 3 and Figure 4, even though the equilibrium probabilities of playing R are nearly opposite to one another. That is to say, the individual frequencies of play were in the right direction, with virtually no subjects playing R less than 50% of the time when they should be playing R 87% of the time, and virtually no subjects playing R more than 50% of the time when they should be playing R about 9% of the time.

Figure 3: Large Asymmetry Individual Results

(Left panels – Session 1; Right panels – Session 2; Top panels – Row player, B frequency on horizontal axis; Bottom panels – Column player, R frequency on horizontal axis; vertical red line: equilibrium strategy)

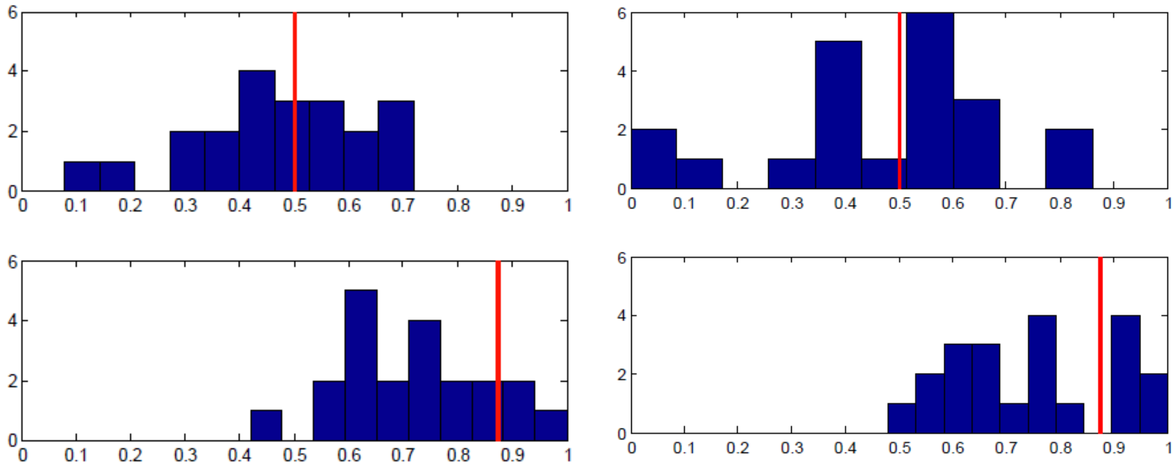
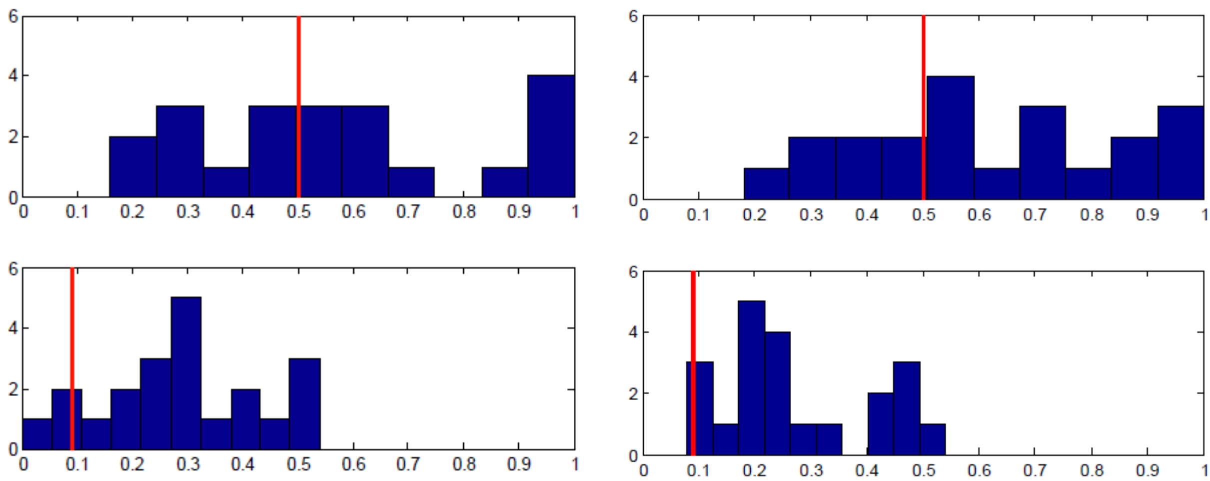


Figure 4: Small Asymmetry Individual Results

(Left panels – Session 1; Right panels – Session 2; Top panels – Row player, B frequency on horizontal axis; Bottom panels – Column player, R frequency on horizontal axis; vertical red line: equilibrium strategy)



We also conducted a post-experiment survey in which we asked subjects a short series of questions about their personal background characteristics, as well as questions about whether they cared about their partners' payoffs, and their experience with the field of game theory.

In terms of prior experience with game theory, we asked them three questions: Have you ever heard of game theory? Have you taken a course on game theory? Do you have any books or academic articles on game theory? Remarkably, *all* of our subjects claimed to have heard of game theory before; we omit this variable from the analysis since it has no explanatory power in behavior in our experiment. 32% of our subjects had taken a course in game theory before. 64% of subjects claimed to have some reading materials on the topic.[3]

Tables 5 and 6 show linear regressions with the individual deviation from the equilibrium mixture, averaged across the Large and Small asymmetry games, as the dependent variable. The results are similar when considering the Large and Small asymmetry games separately. Recall that in our experimental setup, subjects maintained a single role (Row or Column) for both the Large and Small asymmetry treatments.

Table 5: Dependent Variable: Deviation from Equilibrium Mixture (Row players)

	Coefficient	SE
Gender	-0.0348	(0.0446)
Class Year	0.0028	(0.0186)
Game Theory Course	-0.0889*	(0.0513)
Game Theory materials	0.0481	(0.0526)
Other regarding payoffs	0.0668	(0.0516)
Constant	0.1787	(0.1298)

*Standard errors in parentheses. *significant at 10% level; **significant at 5% level*

Table 5, which contains the results for Row players, shows that having taken a game theory course before is significantly (at the 10% level) associated with implementing pure strategy frequencies closer to the theoretical mixture. We do not find a significant effect of any of the explanatory variables for Column players, as seen from Table 6. This may be because Column players experienced significant learning over time (see Table 4), and may be better explained by time dynamics than individual characteristics.

Table 6: Dependent Variable: Deviation from Equilibrium Mixture (Column players)

	Coefficient	SE
Gender	-0.0151	(0.0323)
Class Year	-0.0100	(0.0122)
Game Theory Course	0.0302	(0.0342)
Game Theory materials	-0.0452	(0.0344)
Other regarding payoffs	0.0159	(0.0452)
Constant	0.2437**	(0.0916)

*Standard errors in parentheses. *significant at 10% level; **significant at 5% level*

6. Accounting for Deviations from Equilibrium Play

Although our results do appear much more adherent to equilibrium play than previously found in the literature, a natural next question is how well models of noisy equilibrium play can fit our data

[3] We cannot verify whether those students reporting ownership of game theory reading materials had a correct impression of what 'game theory' is, a possible reason why we find no significant effect of this variable.

compared to previous results. To explore this we consider two possible noisy equilibrium models which have been widely adapted in the literature: ε-equilibrium (Radner, 1980) and Quantal Response Equilibrium (McKelvey and Palfrey, 1995). In our analysis, we closely follow the approach of Levine and Zheng (forthcoming).

6.1 ε-equilibrium

ε-equilibrium represents the payoff noise penalty ε needed to account for the difference between a particular role's empirical expected payoff as determined by the actual frequencies of play, and the payoff from the optimal action given those empirical frequencies. Calculation of the ε-equilibrium provides a lower bound on ε, from which we can infer the range of behavior that would be consistent with such a minimum penalty.

Figure 5: Epsilon Equilibrium Ranges

Top left: Our data, single epsilon for Row and Column players; Top right: Our data, Row and Column specific epsilon; Bottom left: Goeree and Holt (2001), single epsilon for Row and Column players; Bottom right: Goeree and Holt (2001), Row and Column specific epsilon

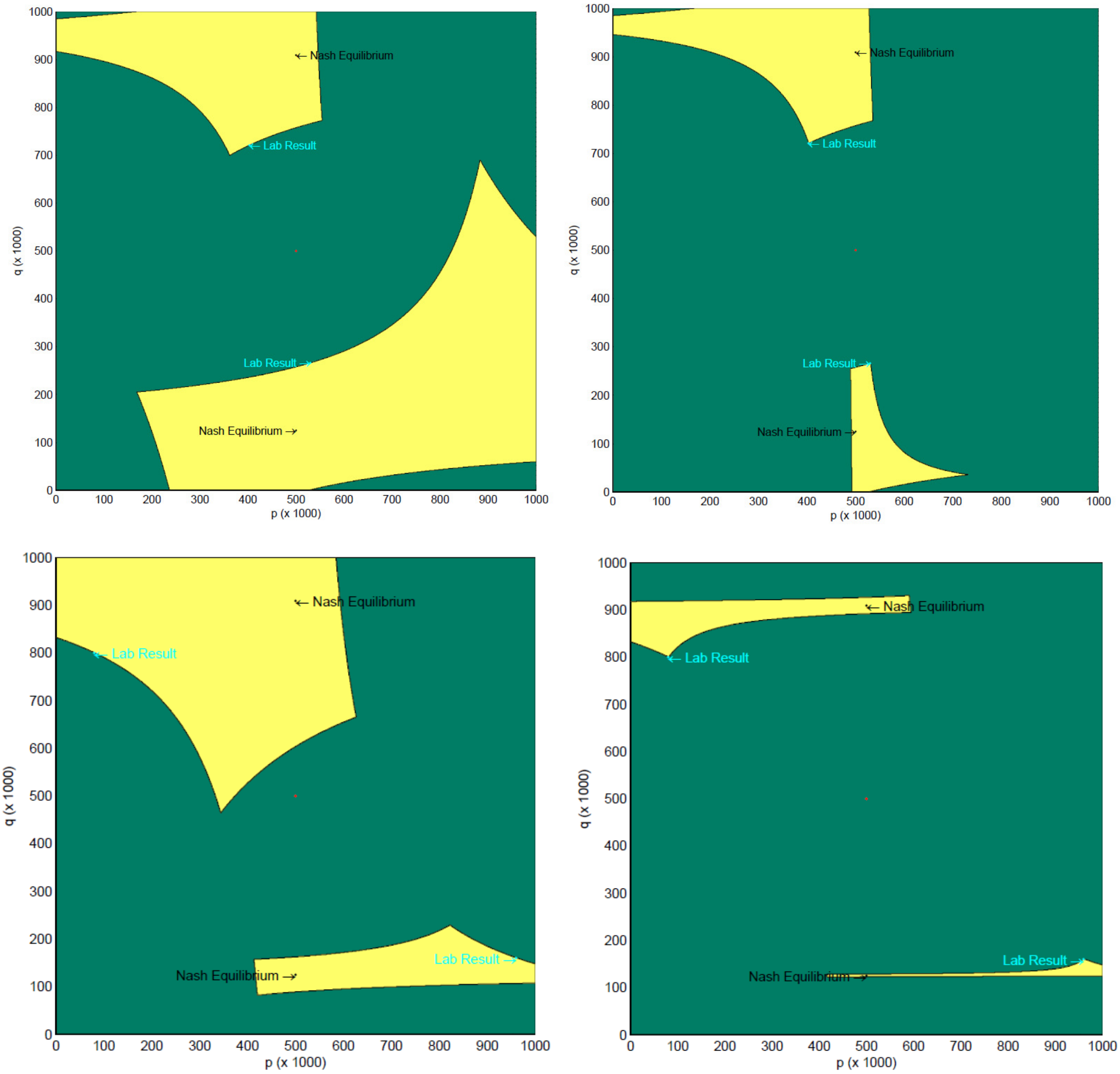


Figure 5 shows the implied range of behavior (in yellow) under the ε-equilibrium assumption for our results (top row) and Goeree and Holt (2001) (bottom row). The Nash equilibrium frequencies and our empirical frequencies are labeled. In each panel, the upper area of yellow corresponds to the Small Asymmetry game and the lower area of yellow corresponds to the Large Asymmetry game. We consider two possible specifications of ε, a common ε for both Row and Column player (left panels), and allowing Row and Column players to have different values of ε (right panels). The common ε assumption is used in Levine and Zheng (forthcoming) and is based on the larger of the two ε values between Row and Column player. The role-specific ε is motivated by the large payoff asymmetry between Row and Column players.

In each panel, the x-axis shows the average randomization of row players, and the y-axis shows the average randomization of column players. From Figure 5, we can see the key difference between our results and Goeree and Holt (2001); our Row players are much closer in their randomization to equilibrium than theirs, while our Column players deviated more than theirs. As the upper right panel shows, the ε-equilibrium with role specific ε allows pinpointing of our empirical result quite precisely. The same precision is not obtained in the case of the common ε assumption (top left panel), due to the fact that column player's ε is quite high.[4]

6.2 Quantal Response Equilibrium

Levine and Zheng (forthcoming) show that Goeree and Holt's results can be accounted for in a Quantal Response Equilibrium (McKelvey and Palfrey, 1995) framework, with players who have altruistic other-regarding preferences. We conduct this same analysis for our subjects, but find that their behavior is better accounted for with a positive *spite* parameter, instead of altruistic preferences towards other players. That is, in allowing for spite, we incorporate in each player's total payoff, a payoff subtraction of the *other* player's payoff, weighted by parameter α (Levine, 1998).

In the Small Asymmetry game, our subjects can be explained by QRE with spite parameter 0.2, as illustrated in Figure 6. We note the contrast with the Goeree and Holt (GH) result, which conversely requires positive altruism in this framework to explain the empirical result.

Our subjects in the Large Asymmetry game require a much higher spite parameter to explain the result. Figure 7 shows QRE with and without the spite parameter value 1.2. Our subjects' behavior in the Large Asymmetry game is nearly explained by this specification. Once again, the GH result is opposite compared to ours, requiring positive altruism to explain.

These QRE results highlight a key apparent difference between our subjects and the subjects from the US in the previous experiments: Behavior of the US-based subjects in previous experiments, generally corresponds to altruistic preferences over payoffs, while the behavior of our subjects corresponds to spitefulness. That is, our subjects behave as though they were receiving *negative* utility from the other player's gains. Furthermore, the magnitude of spite needed to explain the result is much higher in the Large Asymmetry game than the Small Asymmetry game, suggesting that the greater the payoff possibility, the greater the spite.

[4] Note that ε_{Row} and ε_{Column} can be found as follows using the payoff structure and empirical strategy frequencies:
Large Asymmetry game: $(32*0.2657 + 4*0.7343)*0.53045 + (4*0.2657 + 8*0.7343)*0.44955 + \varepsilon_{Row} \geq 32*0.2657 + 4*0.7343$; $\varepsilon_{Row} \geq 2.1145$; $(4*0.53045 + 8*0.46955)*0.2657 + (8*0.53045 + 4*0.46955)*0.7343 + \varepsilon_{Column} \geq 8*0.53045 + 4*0.46955$; $\varepsilon_{Column} \geq 0.0647$.
Small Asymmetry game: $(4.4*0.72095 + 4*0.27905)*0.4029 + (4*0.72095 + 8*0.27905)*0.5971 + \varepsilon_{Row} \geq 4*0.72095 + 8*0.27905$; $\varepsilon_{Row} \geq 0.3335$; $(4*0.4029 + 8*0.5971)*0.72095 + (8*0.4029 + 4*0.5971)*0.27905 + \varepsilon_{Column} \geq 4*0.4029 + 8*0.5971$; $\varepsilon_{Column} \geq 0.2168$. The common ε specification merely takes the larger of the two roles' ε.

Figure 6: Quantal Response Equilibrium with Low Spite (α = 0.2)

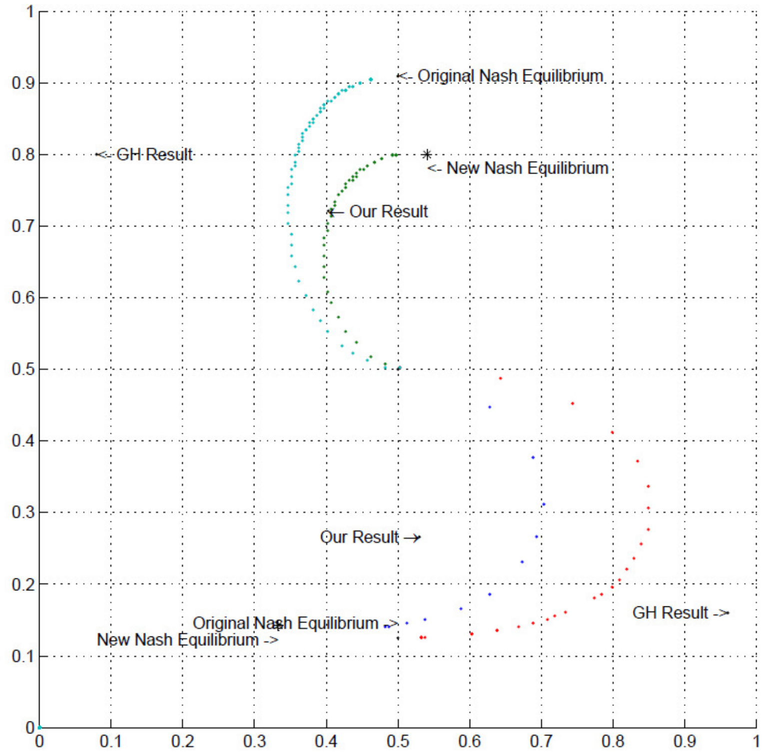
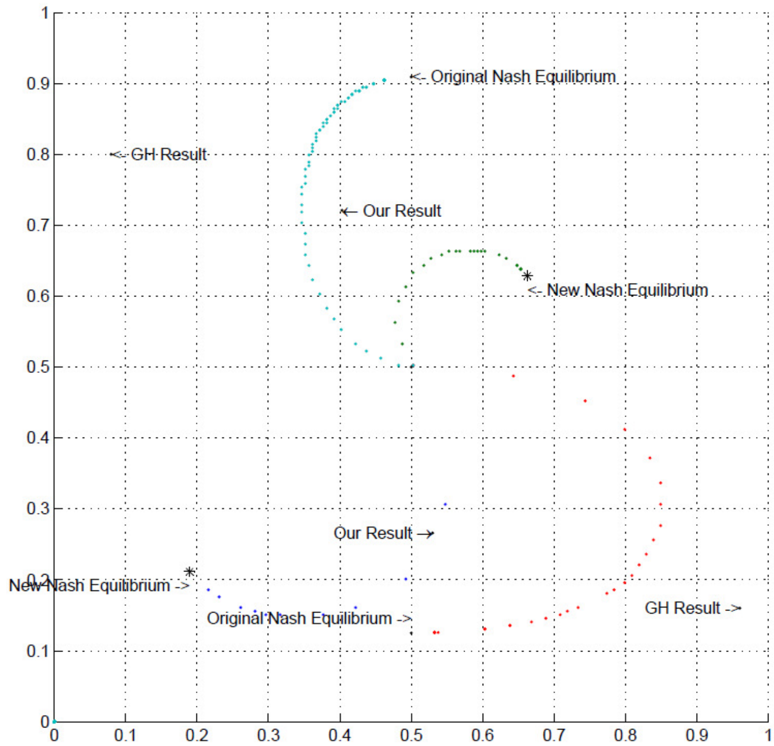


Figure 7: Quantal Response Equilibrium with High Spite (α = 1.2)



Fixed Partner Matching

We also ran sessions with fixed-partner matching over the stage game repetitions. Although most of the literature on experimental asymmetric matching pennies implements random partner re-matching, due to the unique (mixed strategy) equilibrium of the stage game, in our finitely repeated setting, the classical theory makes the same prediction regardless. It is also interesting to check whether subjects' behavior varied when they were dealing with just one opponent throughout.

Consequences of learning in games with unique mixed strategy equilibrium have been analyzed in Crawford (1974) and strategy dynamics have been analyzed empirically in Scroggin (2007). These studies suggest that repeated interactions with a specific opponent are likely to yield different results for behavioral reasons. As in the previous treatments, players' roles were kept fixed throughout each session. Table 7 provides the session summary statistics for the fixed-partner matching treatments.

Table 7: Session Summary

	Session 3	Session 4
Partner Matching	Fixed	Fixed
1st Treatment (Asymmetry)	Large	Small
2nd Treatment (Asymmetry)	Small	Large
Average earnings	76.6	75.6
Standard deviation	11.3	9.3
Minimum earnings	61.2	63.7
Maximum earnings	123.7	98.1
Number of subjects	40	40

The aggregate statistics in Table 8 indicate that for row players, play was close to equilibrium, on average even closer to equilibrium than in the random re-matching treatments. The results for column players tended to deviate a bit further from equilibrium frequencies compared to the random re-matching case. As in the random re-matching treatments, 'B' was more slightly more popular than 'T' among row players in the Small Asymmetry game, but still not near the level of popularity in Goeree and Holt (2001). Table 9 shows that a similar pattern existed when considering just the first two rounds of play.

Table 8: Aggregate Statistics

	Session 3	Session 4
Large Asymmetry (50% B; 87.5% R)	**(51% B; 66% R)**	(51% B; 71% R)
Small Asymmetry (50% B; 9% R)	(54% B; 35% R)	**(57% B; 34% R)**

Note: bold indicates first treatment played

Table 9: Initial Responses (Pure strategy frequencies in first two rounds)

	Session 3	Session 4
Large Asymmetry (50% B; 87.5% R)	**(45% B; 58% R)**	(53% B; 75% R)
Small Asymmetry (50% B; 9% R)	(52% B; 40% R)	**(73% B; 30% R)**

Note: bold indicates first treatment played

Figures 8 and 9 show the individual level results for the Large Asymmetry and Small Asymmetry game respectively. Compared to the random re-matching treatments, the individual frequencies of play tend to display less variance, particularly for the Row players, as seen from the dispersion in frequencies in the top panels of Figure 8 compared to the top panels of Figure 3, and the top panels of Figure 9 compared to the top panels of Figure 4. For the Column players, compared to the case of random re-matching, subjects in the fixed matching treatments tended to center their mixing frequencies farther from the equilibrium prediction. Overall, it appears that fixed partner matching tends to draw play closer to the 50% mark for both player roles, compared to the case of random re-matching.[5]

Figure 8: Large Asymmetry Individual Results

(Left panels – Session 3; Right panels – Session 4; Top panels – Row player, B frequency on horizontal axis; Bottom panels – Column player, R frequency on horizontal axis; vertical red line: equilibrium strategy)

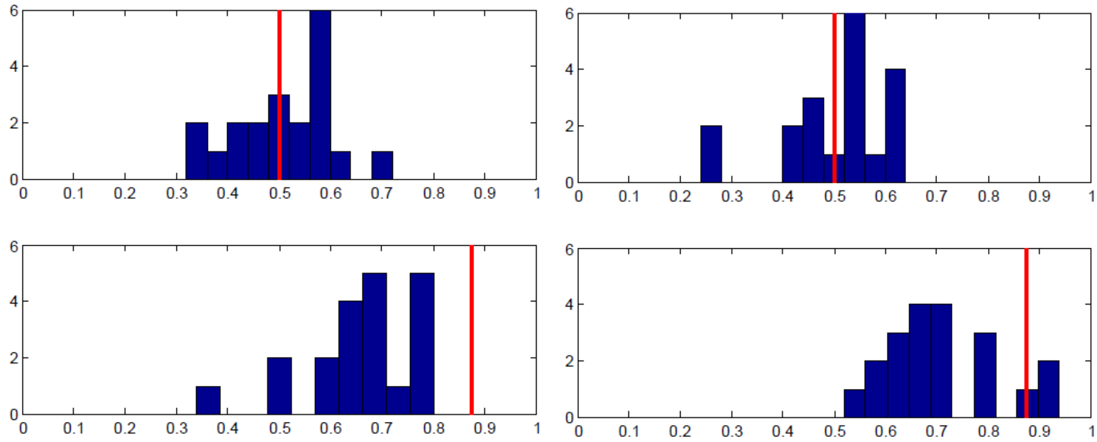
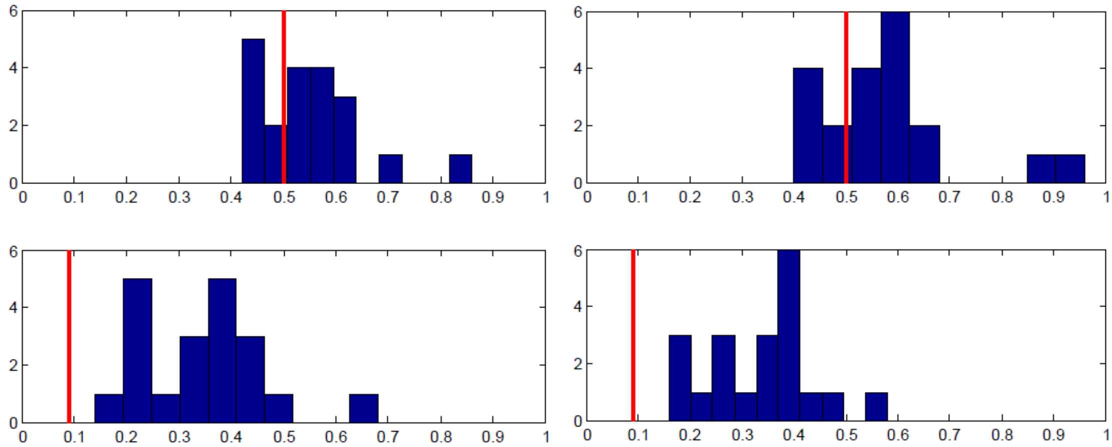


Figure 9: Small Asymmetry Individual Results

(Left panels – Session 3; Right panels – Session 4; Top panels – Row player, B frequency on horizontal axis; Bottom panels – Column player, R frequency on horizontal axis; vertical red line: equilibrium strategy)



7. Conclusions

In this paper, we conducted experiments on the Asymmetric Matching Pennies contradiction posed by Goeree and Holt (2001), which has been robustly confirmed in the literature (Ochs, 1995; McKelvey, Palfrey and Weber, 2000; Goeree, Holt and Palfrey, 2003). Research in the psychology

[5] This makes intuitive sense for Row players, in that suppose a Row player in the random re-matching treatment thinks of himself as one of many possible 'types' in the population, whose aggregated average frequencies of play correspond to the Nash Equilibrium. He may then find it reasonable to play B with 10% likelihood, so long as others play with other likelihoods to average across row players as 50%. In the case of fixed partner matching, this behavioral argument does not hold, which may draw players closer to the 50% equilibrium play. The same pattern does not seem to hold for Column players however, who tend to deviate *more* from equilibrium play in the fixed partner matching treatments.

literature gives us reason to hypothesize that this contradiction may not persist as robustly in non-individualist based cultures as it does in the United States.

Cultural psychology studies suggest that collectivist cultures foster the ability to take other peoples' perspective into account, more than individualist cultures do. For example, Wu and Keysar (2007) found that Chinese subjects performed better than US subjects in a task involving communicating about placing objects, in which they needed to consider a partner's perspective. In a follow up study, Wu, Barr, Gann and Keysar (2013) found that differences in the tendency to take the perspective of others across the two cultures could be attributed to a more rapid and effective correction of egocentric thinking among Chinese subjects. If these tendencies in perspective-taking carry over to non-cooperative game settings, Chinese subjects should adopt mixed strategy equilibrium reasoning more readily than the US subjects in previous studies.

Indeed, our Chinese subjects did not display the contradiction of "own payoff" effects in this unique mixed strategy equilibrium setting. In particular, our Row players did not over-gravitate to the high payoff strategy compared to the equilibrium prediction on average, especially in the Large Asymmetry game. Our Column players, while not adhering as precisely to the equilibrium prediction on average compared to Row players in our study, showed significant adjustment towards the direction of equilibrium over time. Due to the prevalence of results in the literature which support the Matching Pennies contradiction, which include both repetition and a visible history of play as we do (Ochs, 1995; McKelvey, Palfrey and Weber, 2000; Goeree, Holt and Palfrey, 2003), we cannot infer that any feature our experimental design is responsible for this difference in results.

When we consider the concept of ε-equilibrium, particularly in the case of role-specific ε, our results are more precisely pinpointed on the Row players' dimension within the ε-equilibrium framework than previous experimental results. This is due to the fact that our Row players adhered quite closely to equilibrium frequencies, while Column players deviated significantly.

In the Quantal Response Equilibrium framework allowing for altruism or spite, this corresponded to a small positive spite parameter to match our Small Asymmetry results, and a large positive spite parameter to match our Large Asymmetry results. This result is in distinct contrast to the results of previous experiments analyzed within the Quantal Response Equilibrium framework with possible altruism or spite, where which deviations were in the direction of altruism (Levine and Zheng, forthcoming). In other words, our subjects behaved in a manner consistent with receiving negative utility from other player's positive payoffs.

An important factor which we believe may contribute in generating these results is the competitive nature of modern Chinese society. It is widely known that students in China face intense academic competition from a young age, at each stage of education. Students have been conditioned to compete with their peers for a high score. This may help to eliminate phenomena such as altruistic preferences over opponents' payoffs, or desires to take trial-and-error based approaches. This mentality may have led our subjects to take the game and its payoff consequences more seriously than the subjects in the US. It may also be related to the prevalent interest in game theory, as self-reported by our subjects.

We can see several directions for future research. First, while the differences between Chinese subjects' performance in the asymmetric matching pennies games and that of US subjects is striking, more work is needed to verify the hypothesis that Chinese subjects pay greater attention to their opponents' incentives than Western subjects in non-cooperative games. In the context of behavioral 'anomalies' in game theory, some of the findings other than the asymmetric matching pennies contradiction may be robust to Chinese populations. Second, we have only tested the game on Chinese subjects, and not on subjects from collectivist but non-Chinese societies. More work is needed to test the robustness of our findings across other collectivist cultures. Currently, we can only assert a difference in the behavior of US and Chinese subjects. Finally, we believe that further work using other methodologies can be conducted on these same games in China to more precisely pinpoint the perspective-taking hypotheses. Methodologies involving biological indicators or search tracking may be one such direction. We leave these directions to future research.

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Appendix: Experiment Instructions (translated from Chinese); Content in brackets [ ] and ordering of Game 1 and Game 2, varies depending on treatment.

Thank you for participating in this experiment. Please note that you may not communicate with other participants throughout the experiment. If you have questions, you can feel free to ask. Please raise your hand, the experimenter will go to you to answer your question individually.

Our experiment will be conducted in two phases. The first stage is the main part of the experiment, and the second stage is a brief survey. The following document describes only the first stage. When the first stage of the experiment is over, we will introduce and carry out the second phase.

In the experiment, you will participate in two different games. Your earnings will depend on your own choices and the choices made by other participants in the game.

Assigning player types:
In each game, participants are divided into two roles: Player 1 and Player 2. Your role will be determined by the computer randomly in accordance with a uniform probability distribution, and will be displayed on your computer screen. Once determined, your role within the game remains unchanged.

Each game has 50 rounds. *[In each round of the game, the computer will randomly match you to another participant in the opposite role anonymously. In each round of the game, the program ensures that there is an equal chance for you to be matched with any member of the opposite role. (ie. if you are Player 1, in each round you will be randomly matched to a Player 2. If you are Player 2, in each round you will be randomly matched to a Player 1.)*

When Game 1 is completed, we will begin Game 2. The random role selection process and matching of participants are the same in Game 2 as in Game 1.]

[You will be matched to a participant in the opposite role, and will remain matched to this participant for all 50 rounds. (ie. if you are Player 1, you will be randomly matched to a Player 2 in the first round, and you will continue playing with this participant for all 50 rounds. If you are Player 2, you will be randomly matched to a Player 1 in the first round, and you will continue playing with this participant for all 50 rounds.)]

Game 1:
Each participant will play 50 rounds of the following game, with another participant determined in accordance with the previously discussed matching process.

In each round of the game, player 1 has two options: T and B; Player 2 has two choices: L and R.

Each player chooses one of their options, and the possible earnings are as follows:

(i) if Player 1 chooses T and Player 2 chooses L, then Player 1 earns 3.20 yuan, and Player 2 earns 0.40 yuan.
(ii) if Player 1 chooses B and Player 2 chooses L, then Player 1 earns 0.40 yuan, and Player 2 earns 0.80 yuan.
(iii) if Player 1 chooses T and Player 2 chooses R, then Player 1 earns 0.40 yuan, and Player 2 earns 0.80 yuan.
(iv) if Player 1 chooses B and Player 2 chooses R, then Player 1 earns 0.80 yuan, and Player 2 earns 0.40 yuan.

In the beginning of each round, you will observe in the center of the screen, the number of rounds in this game, your earnings and your role. Meanwhile, on the left of the screen will display the game's round by round history (number of rounds played, your role, the other player’s role, your choice, the other player’s choice, your earnings, the other player’s earnings).

In the bottom of the same screen, you need to follow the prompts, and enter your choice (if you are Player 1, select T or B; If you are Player 2, select L or R). After making your choice, please click on the "OK" button to confirm. Once you confirm your selection, it can no longer be changed. You may not return to the previous page.

After you have made your selection, the computer will display a waiting screen. Please be patient and wait for the other players to make their choices. After all participants enter their choice, the screen will show you and your opponent's choices and the corresponding earnings in the current round. Please click on the "Continue" button to wait for the next round of the game.

This process will continue until all participants have completed 50 games. Please wait for the experimenters to start the program in order to start Game 2.

Game 2:
Each participant will play 50 rounds of the following game, with another participant determined in accordance with the previously discussed matching process.

In each round of the game, players have two options 1: T and B; Player 2 has two choices: L and R.

Each player chooses one of their options, and the possible earnings are as follows:

(i) if Player 1 selects T and Player 2 selects L, then Player 1 gets 0.44 yuan, and Player 2 gets 0.40 yuan.
(ii) if Player 1 selects B and Player 2 selects L, then Player 1 gets 0.40 yuan, and Player 2 gets 0.80 yuan.
(iii) if Player 1 selects T and Player 2 selects R, then Player 1 gets 0.40 yuan, and Player 2 gets 0.80 yuan.
(iv) if Player 1 selects B and Player 2 selects R, then Player 1 gets 0.80 yuan, and Player 2 gets 0.40 yuan.

In the beginning of each round, you will observe in the center of the screen, the number of rounds in this game, your earnings and your role. Meanwhile, on the left of the screen will display the game's round by round history (number of rounds played, your role, the other player's role, your choice, the other player's choice, your earnings, the other player's earnings).

In the bottom of the same screen, you need to follow the prompts, and enter your choice (if you are Player 1, select T or B; If you are Player 2, select L or R). After making your choice, please click on the "OK" button to confirm. Once you confirm your selection, it can no longer be changed. You may not return to the previous page.

After you have made your selection, the computer will display a waiting screen. Please be patient and wait for the other players to make their choices. After all participants enter their choice, the screen will show you and your opponent's choices and the corresponding earnings in the current round. Please click on the "Continue" button to wait for the next round of the game.

This process will continue until all participants have completed 50 rounds, ending Game 2. Please wait for the experimenters to start the program in order to start the survey.

Payments:
You will receive monetary earnings from the experiment. You will receive 10 yuan show-up fee, in addition to the sum of earnings you obtain from each round of games 1 and 2.

Upon the end of the experiment, the computer will automatically calculate the total earnings obtained by each participant.

Before the experiment begins:
Please take a few minutes to read the description of the experiment again at your own pace, and then we will begin.

If you have any questions, please raise your hand now. If there are no questions, we can now begin.