Welfare Implications of a Flexible Retirement Policy

Zhenhua Feng±   Jaimie W. Lien†   Jie Zheng‡

Initial Version: May 31st, 2017
This Version: August 1st, 2017

Abstract:[1]

Facing lengthening lifespans and economic concerns, workers and governments are increasingly considering the possibility of delayed retirement ages. However, the postponement of retirement may not be universally feasible, since not all workers may be willing and able to continue working past the standard retirement age, due to health status and other factors. We model this uncertain retirement problem in an overlapping generations general equilibrium framework with flexible retirement, where members of the older generation continue working with some probability, and otherwise retire. Comparing the policies of flexible retirement and mandatory retirement, we find that the consumption and welfare consequences depend largely on the labor intensity of the production function. Higher labor intensity of production tends to yield favorable social welfare results for the flexible retirement policy compared to the mandatory policy. We discuss policy insights and possible implications in China and other demographically shifting countries.

JEL Codes: H3, I1, J1

Keywords: retirement policy, welfare economics, overlapping generations, demographic trends

[1] ± Feng: Institute of Economics, Tsinghua University, Beijing, China, fengzhh.13@sem.tsinghua.edu.cn; † Lien: Department of Decision Sciences and Managerial Economics, The Chinese University of Hong Kong, Hong Kong, jaimie.academic@gmail.com ; ‡ Zheng: Department of Economics, School of Economics and Management, Tsinghua University, Beijing, China, jie.academic@gmail.com.

Delaying the retirement age is one of the most intensely debated, yet seemingly inevitable policy considerations in modern economies. As life expectancies increase while pension and social security systems face growing financial pressures of decreasing younger cohort sizes, increasing the retirement age of workers is a commonly proposed solution. However, asking citizens to delay their retirement plans is often a controversial and in some cases, a vehemently opposed policy. Many workers have already planned for and looked forward to their retirement during much of their lives based on the current retirement age. Conditional on life expectancy, delayed retirement means fewer years spent enjoying the retired lifestyle including their retirement benefits. In addition, a key concern is that workers may not be willing and able to continue working past their anticipated retirement age due to health reasons and other circumstances.

In China, where increasing the retirement age is already part of an official plan to reform the government's pension system, an estimated 68% (People's Daily) to 95% (China Youth Daily, Sohu) of survey respondents are opposed to raising the retirement age.[2] Currently, the retirement age stands at age 60 for male employees (and age 55 for male workers engaging in heavy manual labor), and age 50 for female employees (and 45 for female workers engaging in heavy manual labor). While this retirement age may appear low by the standards of industrialized countries, much of China's labor force engages in high work intensity, including factory work and other manual labor. 60 percent of respondents believed that they would be physically incapable of working by the proposed retirement age of 65 (China Youth Daily, Sohu). At the same time over 73 percent of respondents supported the idea of flexible (early or late) retirement instead of a one-size fits all policy (People's Daily).

A flexible retirement policy is a feasible option which could potentially satisfy some of workers' retirement concerns. We develop a model which examines the consequences of a flexible retirement policy in a general equilibrium framework. In our model, young workers provide full labor supply and choose their savings level. During workers' elderly stage of life, if they are healthy then they can work, but if they are unhealthy they retire from the labor force. Since young workers know the probability of being healthy when old, but do not know their health status ahead of time, the flexible retirement scenario induces lower saving compared to the case of mandatory retirement, which is the status quo in many countries including China.

The general equilibrium approach allows us to analyze one of the other most frequently raised concerns regarding delayed retirement of the older generation: the labor market consequences for young workers. Our model finds that indeed, the wage rate in the labor market as a whole is lower under the flexible retirement policy than under the mandatory scheme.

With the flexible retirement plan inducing both lower savings of young workers and lower wages, a natural question is under which retirement scheme are workers better off? On the one hand, lower wages and savings implies less capacity to consume over the lifetime, but on the other hand, under flexible retirement, the option of working until a later age and continuing to receive wages yields a possibility of higher overall consumption. A comparison of consumption and lifetime utilities under flexible and mandatory retirement schemes reveals that the superiority of each retirement scheme depends largely on the production technology of the economy.

A labor intensive production process tends to favor a flexible retirement scheme in terms of maximizing workers' consumption and lifetime utility, whereas a capital intensive production process tends to favor the mandatory retirement. The effect of the intensity of factors of production is substantially stronger than the effect of the likelihood of being healthy in old age. This result could serve as a benchmark guidance for countries seeking to choose a retirement policy for their citizens. Furthermore, it suggests that in the case of China, which has been a labor intensive economy, a

[2] See http://news.xinhuanet.com/english/china/2013-11/01/c_132850992.htm and http://www.ecns.cn/2013/09-02/79395.shtml.

flexible retirement policy may have significant advantages to workers.

Our study contributes to the literature on the challenge of retirement policies around the world in dealing with demographic changes. Among them, Kalemi-Ozcan and Weil (2010) study the optimal retirement problem of a population facing decreasing mortality. Under high mortality, savings is risky since the returns on saving may not be able to be realized, whereas lowering mortality can properly induce savings for retirement. Heijdra and Romp (2009) study a problem in which citizens in a small open economy choose their retirement age with information about wage profiles, taxation and pension programs, and analyze the early retirement feature of many pension systems. Philippe and Pestieau (1999) analyze the dynamics of the social security system, proposing an explanation of the increasing financial difficulties of the pay-as-you-go system, and showing that early retirement could be socially beneficial. Also focusing on the social security system, Imrohoroglu, Imrohorglu and Joines (1995) analyze the welfare consequences of unfunded social security in an overlapping generations framework with mortality and income risks incorporated.

Our research shares with many of these studies the overlapping generations framework (Samuelson, 1958; see also Weil 2008 for an overview) for analysis, and the objective of analyzing worker's retirement problem. Our set up differs from many of these studies in our focus on the case of autarky, or in other words, a retirement policy with purely self-savings but no social security program. We analyze the case of autarky as a benchmark and basis for understanding the general equilibrium consequences of flexible versus mandatory retirement schemes. Our framework is also unique to our knowledge, in its comparison of the mandatory retirement policy which reflects the current reality in countries like China, to a more flexible scheme which allows for workers' willingness and ability to continue in the labor market, which is an often suggested policy proposal, particularly from the perspective of workers.

The paper proceeds as follows: Section 2 describes the model set up and solves the steady state equilibrium under different policies; Section 3 conducts welfare analysis and provides policy implications; Section 4 concludes and discusses future research directions.

2. Model

We consider a two-period overlapping generations model under a production economy. Agents have two stages of life: a young stage in which they are willing and able to participate in the labor market for sure; and an elderly stage in which there are two different possible policies and outcomes. In the mandatory retirement policy, the agent must retire and is not allowed to work during the elderly stage. In the flexible retirement policy, the agent can work if he is healthy, and cannot work if he is unhealthy, where health status is determined by chance.

Each agent has a per period utility function $u(c) = \ln c$ and a time discount factor β. Note that utility depends only on consumption and not on leisure, so that the agent will spend all his time working, if possible. The agent will make a consumption and savings decision in his youth, which will at least partially determine the amount of money available to him in the elderly stage.

Production in the economy is given by the Cobb-Douglas production function, with constant returns to scale $Y = F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$, where L denotes labor, K denotes capital, and A denotes technological progress. The savings of agents are converted to capital at unit conversion in their stage of youth, and are returned to agents in the elderly stage along with a return on the capital investment. From the firm's perspective, there is a one period delay in receiving its capital stock, over which it depreciates by discount factor δ. The population in period t is denoted by N_t where the population evolves according to $N_{t+1} = (1+n)N_t$. Note that we focus on the case of autarky, so there is no mandatory savings or pension program in place.

2.1 Flexible Retirement Policy

We first consider the case of a flexible retirement policy. In such policy, an agent is allowed to work when he is healthy and cannot work when he is unhealthy. Suppose that a young agent is healthy with probability 1, and an old agent is healthy with probability $p\ (0 < p \le 1)$ and unhealthy with probability $1-p$. Workers supply full labor whenever healthy since there is no utility of leisure. Thus, at period t, the total labor supply is $L_t = N_t + pN_{t-1}$.

2.1.1 The Workers' Problem

Generation-t's problem is to maximize their lifetime expected utility:

$$Max_{c_t^t, c_{t+1,h}^t, c_{t+1,u}^t} \ Eu = \ln c_t^t + \beta(p(\ln c_{t+1,h}^t) + (1-p)\ln c_{t+1,u}^t)$$

$$\begin{aligned} s.t.\ & c_t^t = w_t - s_t, \\ & if\ healthy, c_{t+1,h}^t = w_{t+1} + R_{t+1}s_t; \\ & if\ unhealthy, c_{t+1,u}^t = R_{t+1}s_t; \\ & c_t^t, c_{t+1,h}^t \ge 0, c_{t+1,u}^t \ge 0. \end{aligned}$$

where the superscript refers to the period in which the worker is born, and the subscript refers to the current period. c denotes consumption, s denotes savings, w denotes wage, $R = r + \delta$ is the gross interest rate, and β is the worker's discount factor between young and elderly stages of life. Health status is denoted by u for unhealthy and h for healthy.

Throughout the model, note that lower case symbols represent individual level variables and upper case letters are aggregate variables.

The first order condition of the consumer's utility maximization problem is:

$$-\frac{1}{w_t - s_t} + \beta\left(\frac{1-p}{s_t} + \frac{pR_{t+1}}{R_{t+1}s_t + w_{t+1}}\right) = 0 \qquad (1)$$

which yields the following two solutions for the savings of young workers: $s_{t1}^* = \frac{1}{2}\frac{\sqrt{G} + \beta(R_{t+1}w_t - (1-p)w_{t+1}) - w_{t+1}}{(1+\beta)R_{t+1}}$ and $s_{t2}^* = \frac{1}{2}\frac{-\sqrt{G} + \beta(R_{t+1}w_t - (1-p)w_{t+1}) - w_{t+1}}{(1+\beta)R_{t+1}}$, where

$$G = (1+(1-p)\beta)^2 w_{t+1}^2 + 2\beta((1-p)\beta + 1 - 2p)R_{t+1}w_t w_{t+1} + \beta^2 R_{t+1}^2 w_t^2 .$$

Comparing the two solutions, we find that $\beta(R_{t+1}w_t-(1-p)w_{t+1})-w_{t+1}\le\sqrt{G}$ because $G-(\beta(R_{t+1}w_t-(1-p)w_{t+1})-w_{t+1})^2=4\beta R_{t+1}w_tw_{t+1}(1-p)(\beta+1)\ge 0$. This implies $s_{t1}^*\ge 0, s_{t2}^*\le 0$ .Since a worker consumption when he is old and unhealthy has to be positive due to the log utility assumption, namely $c_{t+1,u}^t=R_{t+1}s_t>0$, the only sensible solution is $s_t^*=s_{t1}^*$.

2.1.2 The Firms' Problem

We now turn to the production side of the economy. Recall that lower case letters denote individual variables, which in this case is on a per-labor basis.

The representative firm's problem is given by the profit maximization problem:

$$Max_{k_t}\ Ak_t^{\alpha}-w_t-r_tk_t-\delta k_t\,,$$

with $y=f(k)=Ak_t^{\alpha}=\dfrac{F(K_t,L_t)}{L_t}$ and $\alpha Ak_t^{\alpha-1}=r_t+\delta=R_{t+1}$, where $k_t\equiv\dfrac{K_t}{L_t}$ represents the capital per labor.

Since the firm's production is constant returns to scale, there will be zero profit such that the firm's output is equal to its wage bill and return on capital paid, that is, $Ak_t^{\alpha}-k_tR_{t+1}=w_t$.

Since s_t^* is a function of R_{t+1},w_t,w_{t+1} which are in turn functions of k_t and k_{t+1}, the expression for the capital stock can be simplified from $k_{t+1}=\dfrac{s_t^*}{1+n+p}$ to $k_{t+1}=\dfrac{s_t^*(k_t,k_{t+1})}{1+n+p}$. Specifically, we have the following condition:

$$k_{t+1}=\frac{(1-\alpha)(1-p+\alpha n+2\alpha p)\beta Ak_t^{\alpha}}{(1+n+p)(1+\beta+\alpha p+\alpha n-\beta p+\beta\alpha n+2\beta\alpha p)}.$$

We can then solve for the firm's steady stage capital stock by setting $k_{t+1}=k_t$.

There are two potential steady-states:

$$\hat{k}_1=\left(\frac{A\beta(1-\alpha)(1-p+\alpha n+2\alpha p)}{(1+n+p)(1+\beta+\alpha p+\alpha n-\beta p+\beta\alpha n+2\beta\alpha p)}\right)^{\frac{1}{1-\alpha}},\ \text{and}\ \hat{k}_2=0.$$

Of the two steady state solutions, we examine which of them is stable, and find that

$\left.\dfrac{dk_{t+1}}{dk_t}\right|_{\hat{k}_2=0}=+\infty>1$, so we know $\hat{k}_2$ is unstable.

$\left.\dfrac{dk_{t+1}}{dk_t}\right|_{\hat{k}_1}=\alpha<1$, therefore $\hat{k}_1$ is stable.

To summarize, under the flexible retirement policy, in the steady state, the capital per labor is

$\hat{k}_p = (\frac{A\beta(1-\alpha)(1-p+\alpha n+2\alpha p)}{(1+n+p)(1+\beta+\alpha p+\alpha n-\beta p+\beta\alpha n+2\beta\alpha p)})^{\frac{1}{1-\alpha}}$, the individual saving of a young worker is $\hat{s}_p = (1+n+p)\hat{k}_p$, the individual saving of an old worker regardless of his health condition is 0, the wage is $\hat{w}_p = A(1-\alpha)(\hat{k}_p)^{\alpha}$, the gross interest rate is $\hat{R}_p = \alpha A(\hat{k}_p)^{\alpha-1}$, the individual consumption of a young worker is $\hat{c}_{yp} = A(1-\alpha)(\hat{k}_p)^{\alpha} - (1+n+p)\hat{k}_p$, the individual consumption of a healthy old worker is $\hat{c}_{op_h} = A((1-\alpha)+\alpha(1+n+p))(\hat{k}_p)^{\alpha}$, the individual consumption of an unhealthy old worker is $\hat{c}_{op_i} = A\alpha(1+n+p)(\hat{k}_p)^{\alpha}$.

2.2 Mandatory Retirement Policy

We now examine the workers' and firm's problem under the case of mandatory retirement. In other words, when workers reach the elderly stage, they retire regardless of their health status. This reflects the current policy in China and some other countries. In Section 3, we then compare the Flexible and Mandatory Retirement Policies.

Compared to the Flexible Retirement Policy, the workers' problem is simplified in that the future is certain when the worker is young. Consumption when elderly is no longer dependent on health status. The Mandatory Retirement Policy can also be understood as a special case of the Flexible Retirement Policy where the probability of being unhealthy when elderly is equal to one ($p=0$).

2.2.1 The Workers' Problem

Generation-t's problem is to maximize their lifetime utility:

$$Max_{c_t^t, c_{t+1}^t} \ \ln c_t^t + \beta \ln c_{t+1}^t$$

$$\begin{aligned} s.t.\ & c_t^t = w_t - s_t; \\ & c_{t+1}^t = R_{t+1} s_t; \\ & c_t^t, c_{t+1}^t \geq 0. \end{aligned}$$

The first order condition for the worker is given by:

$$-\frac{1}{w_t - s_t} + \frac{\beta}{s_t} = 0 \qquad (2)$$

From this we obtain the optimal savings plan $s_t^* = \frac{\beta w_t}{1+\beta}$, $s_t^* \in [0, w_t]$.

2.2.2 The Firms' Problem

The firm's problem $Max_{k_t}\ Ak_t^{\alpha} - w_t - r_t k_t - \delta k_t$ is similar to that in the Flexible Retirement Policy in obtaining the zero profit conditions, namely $\alpha A k_t^{\alpha-1} = R_{t+1}$, $Ak_t^{\alpha} - k_t R_{t+1} = w_t$.

The expression for the capital stock is simplified by the certainty faced by workers in the early stage of life, such that $k_{t+1} = \frac{s_t^*}{1+n}$. Since $s_t^* = \frac{\beta w_t}{1+\beta} = \frac{\beta}{1+\beta}(Ak_t^{\alpha} - k_t R_{t+1}) = \frac{\beta}{1+\beta}(Ak_t^{\alpha} - k_t \alpha A k_t^{\alpha-1})$, we can express k_{t+1} in terms of k_t, that is, $k_{t+1} = \frac{A\beta(1-\alpha)k_t^{\alpha}}{(1+n)(1+\beta)}$.

The above condition yields the steady state solutions $\hat{k}_1 = 0, \hat{k}_2 = (\frac{A\beta(1-\alpha)}{(1+n)(1+\beta)})^{\frac{1}{1-\alpha}}$ of which, $\left.\frac{dk_{t+1}}{dk_t}\right|_{\hat{k}_1=0} = +\infty > 1$ is not stable, and $\left.\frac{dk_{t+1}}{dk_t}\right|_{\hat{k}_2} = \alpha < 1$ is stable.

To summarize, under the mandatory retirement policy, in the steady state, the capital per labor is $\hat{k}_r = (\frac{A\beta(1-\alpha)}{(1+n)(1+\beta)})^{\frac{1}{1-\alpha}}$, the individual saving of a young worker is $\hat{s}_r = (1+n)\hat{k}_r$, the individual saving of an old worker regardless of his health condition is 0, the wage is $\hat{w}_r = A(1-\alpha)(\hat{k}_r)^{\alpha}$, the gross interest rate is $\hat{R}_r = \alpha A(\hat{k}_r)^{\alpha-1}$, the individual consumption of a young worker is $\hat{c}_{yp} = A(1-\alpha)(\hat{k}_r)^{\alpha} - (1+n)\hat{k}_r$, the individual consumption of an old worker regardless of his health condition is $\hat{c}_{or} = A\alpha(1+n)(\hat{k}_r)^{\alpha}$, the individual consumption of an unhealthy old worker is $\hat{c}_{op_i} = A\alpha(1+n+p)(\hat{k}_p)^{\alpha}$.

3. Welfare Comparison of Policies

We first compare the solutions for the Flexible Retirement Policy and the Mandatory Retirement Policy, for savings, capital per labor, wage and return on capital. Throughout, we denote the variables associated with the Flexible Retirement Policy with p, and the Mandatory Retirement Policy with r.

Proposition 1: In steady state, saving under the flexible retirement policy is less than saving under the mandatory retirement policy, that is, $\hat{s}_p < \hat{s}_r$.

Proof:

Recall from Section 2.1 that savings under the Flexible Retirement Policy is

$$\hat{s}_p = (1+n+p)(\frac{A\beta(1-\alpha)(1-p+\alpha n+2\alpha p)}{(1+n+p)(1+\beta+\alpha p+\alpha n-\beta p+\beta\alpha n+2\beta\alpha p)})^{\frac{1}{1-\alpha}},$$

while the savings under the Mandatory Retirement Policy (Section 2.2) is given by

$$\hat{s}_r = (1+n)(\frac{A\beta(1-\alpha)}{(1+n)(1+\beta)})^{\frac{1}{1-\alpha}}.$$

Taking the ratio of the savings across the two policies

$$\frac{\hat{s}_p}{\hat{s}_r} = (\frac{(1+\beta)(1-p+\alpha n+2\alpha p)}{1+\beta+\alpha p+\alpha n-\beta p+\beta\alpha n+2\beta\alpha p})^{\frac{1}{1-\alpha}}(\frac{1+n}{1+n+p})^{\frac{\alpha}{1-\alpha}}.$$

Comparing the numerator and denominator of the fraction $\frac{(1+\beta)(1-p+\alpha n+2\alpha p)}{1+\beta+\alpha p+\alpha n-\beta p+\beta\alpha n+2\beta\alpha p}$, we have $(1+\beta+\alpha p+\alpha n-\beta p+\beta\alpha n+2\beta\alpha p)-(1+\beta)(1-p+\alpha n+2\alpha p) = p-\alpha p > 0$. Since we also know $1+n+p > 1+n$, and $1-\alpha > 0$, this implies $\frac{\hat{s}_p}{\hat{s}_r} < 1$, which completes the proof.

The intuition behind the proposition is straightforward. When workers have a possibility of working when elderly under the flexible retirement scheme, the desire for saving when they are young is reduced for the expected utility maximizing worker. On the other hand, when retirement is mandated, the need for saving is certain and thus higher in level than the case of uncertain retirement.

Proposition 2: In steady state, the capital per labor under the flexible retirement policy is less than the capital per labor under the mandatory retirement policy, , that is, $\hat{k}_p < \hat{k}_r$.

Proof:

Recall from Section 2.1 that $\hat{k}_p = \frac{\hat{s}_p}{1+n+p}$ and from Section 2.2 that $\hat{k}_r = \frac{\hat{s}_r}{1+n}$. Since by Proposition 1, $\hat{s}_p < \hat{s}_r$, and we also know $1+n+p > 1+n$, this easily implies $\hat{k}_p < \hat{k}_r$.

The intuition of the result can be understood from the perspective of the increased aggregate labor supply under the flexible retirement policy, as healthy elderly workers remain in the workforce under this scheme. Simultaneously, savings under the flexible retirement policy is lower as stated in Proposition 1, which further reduces the capital per labor in the flexible retirement scheme compared to the mandatory retirement scheme.

The following two corollaries indicate the effect of flexible retirement on the marginal product of labor and marginal product of capital respectively.

Corollary 1: In steady state, the wage rate under the flexible retirement policy is lower than the wage rate under the mandatory retirement policy, that is, $\hat{w}_p < \hat{w}_r$.

Proof:

Since $\hat{w} = A(1-\alpha)\hat{k}^{\alpha}$, and $\hat{k}_p < \hat{k}_r$, it follows immediately that $\hat{w}_p < \hat{w}_r$.

Notice our model does not allow for the possibility of unemployment, so the influx of labor resulting from elderly workforce participation results directly in lower market wages. However, the result reflects one of the frequently mentioned concerns about increasing the retirement age, the potentially adverse consequences for employment conditions for young workers (for a discussion, see Gruber, Milligan and Wise, 2009).

Corollary 2: In steady state, the rate of return on capital under the flexible retirement policy is higher than the wage rate under the mandatory retirement policy, that is $\hat{R}_p > \hat{R}_r$.

Proof:

Since $\alpha A\hat{k}^{\alpha-1} = \hat{R}$ and $\hat{k}_p < \hat{k}_r$, it follows immediately that $\hat{R}_p > \hat{R}_r$.

This result can be understood from the savings behavior of young workers under the respective retirement policies. Under the flexible retirement policy, savings of young workers are reduced as there is a possibility for them to work when elderly, thus reducing the capital stock. Under diminishing returns to capital in the production function, the rate of return on capital is higher under these circumstances.

We now turn to the comparison of consumption behavior under the two retirement policies. Since the analysis of the consumption comparison is complex, we consider the case of no population growth, (meaning $n = 0$) and no impatience (meaning $\beta = 1$). Since the solution is not analytically tractable, we conduct the analysis using simulations. The detailed proof is available upon request and will be made available online from the corresponding author's webpage.

Proposition 3: (Consumption when Young) When the labor intensity of production is high (or the capital intensity of production is low), consumption of young workers is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When the labor intensity of production is low (or the capital intensity of production is high), consumption of young workers is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy

To see this, note that the expression for the ratio of consumption in youth under each policy is

$$\frac{\hat{c}_{yp}}{\hat{c}_{yr}} = \frac{2(\alpha p+1)}{(3\alpha p - p + 2)}\left(\frac{2(2\alpha p - p + 1)}{(3\alpha p - p + 2)(1+p)}\right)^{\frac{\alpha}{1-\alpha}}.$$

Figure 1: Consumption Comparison for Young Workers
Horizontal axis: relative capital intensity of production α
Vertical axis: probability p of being healthy when elderly

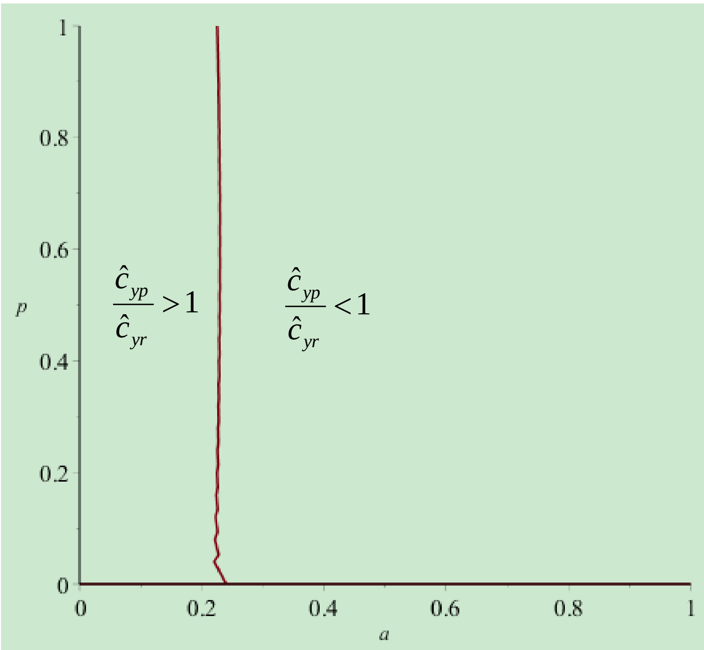


Figure 1 illustrates the cutoff values of capital intensity parameter α and probability of being healthy p, for which consumption of young workers is equalized across retirement policies. The area to the left of the line indicates parameter values for which the Flexible Retirement Policy yields higher consumption of young workers, whereas to the right of the line, the parameters are such that the Mandatory Retirement Policy yields higher consumption.

From the nearly vertical line in Figure 1, we can observe that the question of which policy yields higher consumption of young workers does not rely heavily on the probability of being healthy when elderly. Rather, the consumption consequences of each policy depends nearly entirely on the value of α. When α is low, production is labor intensive. Our simulation (as well as our proof) shows that a labor intensive production economy leads to higher consumption among the young generation under the Flexible Retirement Policy, compared to the Mandatory Retirement Policy.

We now consider the consumption comparison across policies for the elderly. The analysis for the elderly is comprised of the two scenarios, the healthy state and unhealthy state, respectively.

Proposition 4: (Consumption when Elderly) When the labor intensity of production is high (or the capital intensity of production is low), consumption of healthy elderly workers under the Flexible Retirement Policy is higher than consumption of unhealthy elderly workers under the Flexible Retirement Policy,which is higher than consumption of elderly workers under the Flexible Retirement Policy than under the Mandatory Retirement Policy: $\hat{c}_{op_h} > \hat{c}_{op_i} > \hat{c}_{or}$;

When the labor intensity of production is low (or the capital intensity of production is high) and the probability of being healthy is relatively low, consumption of healthy elderly workers under the Flexible Retirement Policy is higher than consumption of elderly workers under the Mandatory Retirement Policy, which is higher than consumption of unhealthy elderly workers under the Flexible Retirement Policy: $\hat{c}_{op_h} > \hat{c}_{or} > \hat{c}_{op_i}$;

When the labor intensity of production is low (or the capital intensity of production is high) and the probability of being healthy is relatively high, consumption of elderly workers under the Mandatory Retirement Policy is higher than consumption of healthy elderly workers under the Flexible Retirement Policy, which is higher than consumption of unhealthy elderly workers under the Flexible Retirement Policy: $\hat{c}_{or} > \hat{c}_{op_h} > \hat{c}_{op_i}$.

When the elderly are healthy, the consumption ratio across retirement policies is given by

$$\frac{\hat{c}_{op_h}}{\hat{c}_{or}} = \frac{\left(\frac{2(2\alpha p - p + 1)}{(3\alpha p - p + 2)(1+p)}\right)^{\frac{\alpha}{1-\alpha}}(\alpha p + 1)}{\alpha};$$

When the elderly are unhealthy, the consumption ratio across retirement policies is given by

$$\frac{\hat{c}_{op_i}}{\hat{c}_{or}} = \left(\frac{2(2\alpha p - p + 1)}{3\alpha p - p + 2}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{1+p}\right)^{\frac{2\alpha - 1}{1-\alpha}},$$

Figure 2 shows the values of parameters p and α for which consumption is equal across the two retirement policies.

The blue line indicates the case of being unhealthy when elderly. Similarly to the case of consumption when young, the relative consumption advantage across policies depends primarily on the labor intensity of production. When production is labor intense, or in other words α is low, consumption under the Flexible Retirement Policy is higher than that under the Mandatory

Retirement Policy. When production is relatively capital intense, Mandatory Retirement carries the higher consumption level for the unhealthy elderly citizen.

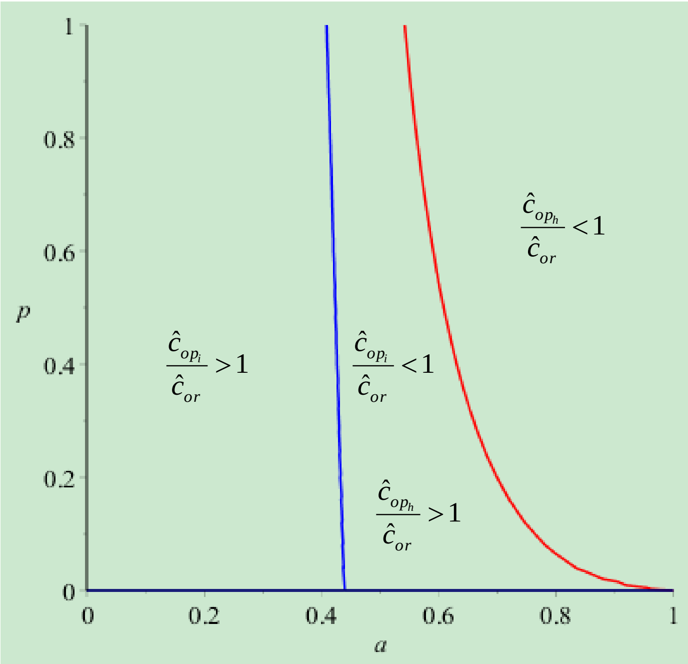
The red line shows the case of being healthy when old. In this case, the elderly citizen is still working in the labor market, and potentially receiving both wage income and returns on capital investment from the young stage of his or her life. Here, we can observe a degree of tradeoff between the parameters p and α. Consumption in the Flexible Retirement Policy is greater than that in the Mandatory Retirement Policy for parameter values to the left (and below) of the red line. The red line shows that particularly for low likelihoods of being healthy enough to work when old, a lower likelihood of health in old age allows for greater capital intensity of production in order to yield the same consumption level. In other words, the range of production functions (in the Cobb-Douglas class) for which Flexible Retirement is better for the healthy elderly person is increasing as the probability of being healthy shrinks.

Finally, Figure 2 demonstrates that the consumption consequences of each retirement policy are more generally sensitive to the production technology in the case of the unhealthy worker compared to the case of the healthy worker. This is intuitive since healthy workers will still be in the labor force when elderly, and their consumption advantage in the Flexible Retirement Policy is easier to achieve, regardless of the specifics of production.

Figure 2: Consumption Comparison for Healthy (red line) and Unhealthy (blue line) Elderly

Horizontal axis: relative capital intensity of production α

Vertical axis: probability p *of being healthy when elderly*



Combining the consumption results for young workers and elderly workers under each health status, we can consider the aggregated consumption across these situations for each retirement policy. To do this, we compare the consumption of the currently living generations under the Flexible

Retirement Policy to the consumption of currently living generations under the Mandatory Retirement Policy. The result, once again by simulation, yields Proposition 5.

Proposition 5 (Aggregate Consumption): There exist $\alpha^{*}_{Ac}, \alpha^{**}_{Ac} \in (0,1)$, $\alpha^{*}_{Ac} < \alpha^{**}_{Ac}$. For $\alpha > \alpha^{**}_{Ac}$, the aggregate consumption under the flexible retirement policy is less than the aggregate consumption under the mandatory retirement policy; for $\alpha < \alpha^{*}_{Ac}$, the aggregate consumption under the flexible retirement policy is more than the aggregate consumption under the mandatory retirement policy,

$$\frac{\hat{C}_{pt}}{\hat{C}_{rt}} = \frac{p\hat{c}_{op_h} + (1-p)\hat{c}_{op_l} + \hat{c}_{yp}}{\hat{c}_{or} + \hat{c}_{yr}}$$

$$= \frac{2(2\alpha^2 p + 3\alpha p^2 - p^2 + \alpha + 2p + 1)\left(\frac{2(2\alpha p - p + 1)}{(3\alpha p - p + 2)(1+p)}\right)^{\frac{\alpha}{1-\alpha}}}{(3\alpha p - p + 2)(\alpha + 1)}.$$

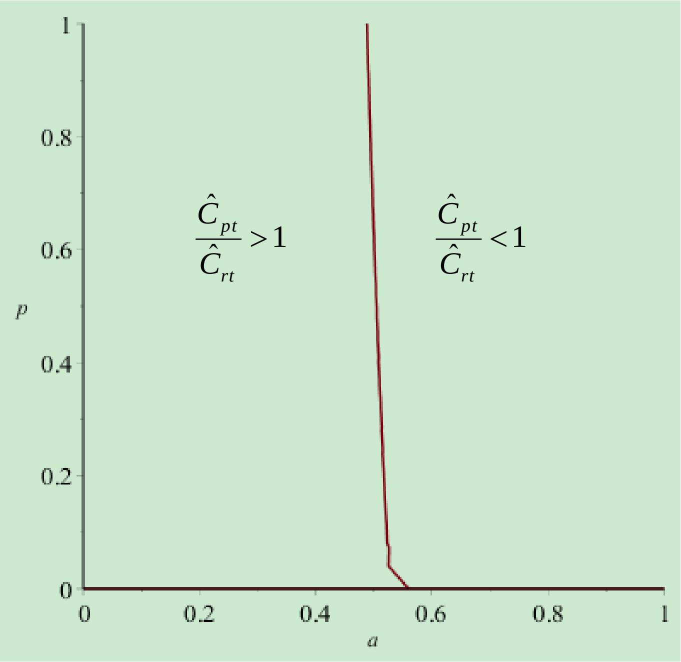
$$= \frac{2(2\alpha^2 p + 3\alpha p^2 - p^2 + \alpha + 2p + 1)\left(\frac{2(2\alpha p - p + 1)}{1+p}\right)^{\frac{\alpha}{1-\alpha}}}{(3\alpha p - p + 2)^{\frac{1}{1-\alpha}}(\alpha + 1)}$$

Figure 3 shows that similarly to the results for individual generations, the consumption consequences of the retirement policies depend primarily on the production function parameter. For more capital intensive production, the mandatory retirement policy yields higher consumption, while for the more labor intensive production, the flexible retirement policy is more favorable.

Figure 3: Aggregate Consumption Comparison

Horizontal axis: relative capital intensity of production α

Vertical axis: probability p *of being healthy when elderly*



Thus far, our analysis has focused on the consumption consequences of each retirement scheme, but it can be easily extended to draw welfare insights for workers. We consider workers' lifetime utilities based on the original assumption of log utility. The result is stated in Proposition 6, and can be seen most straightforwardly from Figure 4, which is analogous to the previous figures, however making the comparison in utility terms.

Proposition 6 (Lifetime Utility): There exist $\alpha_U^*, \alpha_U^{**}, \alpha_U^{***} \in (0,1)$, $\alpha_U^* < \alpha_U^{**} < \alpha_U^{***}$. For $\alpha > \alpha_U^{***}$, for larger p, the lifetime utility under the flexible retirement policy is more than the lifetime utility under the mandatory retirement policy. For $\alpha_U^{**} < \alpha < \alpha_U^{***}$, the unhealthy lifetime utility under the flexible retirement policy is less than the lifetime utility under the mandatory retirement policy while the healthy lifetime utility under the flexible retirement policy is more than the lifetime utility under the mandatory retirement policy. For $\alpha < \alpha_U^*$, the lifetime utility under the flexible retirement policy is more than the lifetime utility under the mandatory retirement policy.

Under the flexible retirement policy, if an agent is unhealthy when old, his lifetime utility is

$$\hat{U}_{pi} = \ln(A(1-\alpha)\hat{k}_p^{\alpha} - (1+n+p)\hat{k}_p) + \beta\ln(\alpha A\hat{k}_p^{\alpha}(1+n+p)),$$

Under the mandatory retirement policy, his lifetime utility is

$$\hat{U}_r = \ln(A\hat{k}_r^{\alpha}(1-\alpha) - (1+n)\hat{k}_r) + \beta\ln(\alpha A\hat{k}_r^{\alpha}(1+n)),$$

Assuming $n=0, \beta=1$, we have

$$\begin{aligned}(1-\alpha)(\hat{U}_{pi} - \hat{U}_r) &= (1-\alpha)(\ln((1-\alpha)A - (1+p)\hat{k}_p^{1-\alpha}) + 2\alpha\ln(\hat{k}_p) + \ln(1+p) - \ln((1-\alpha)A - \hat{k}_r^{1-\alpha}) - 2\alpha\ln(\hat{k}_r)) \\ &= \ln(\frac{(4\alpha p - 2p + 2)^{2\alpha}(2\alpha p + 2)^{1-\alpha}(1+p)^{1-3\alpha}}{(3\alpha p - p + 2)^{1+\alpha}})\end{aligned}$$

Under the flexible retirement policy, if an agent is healthy when old, his lifetime utility is

$$\hat{U}_{ph} = \ln(A(1-\alpha)\hat{k}_p^{\alpha} - (1+n+p)\hat{k}_p) + \beta\ln(A\hat{k}_p^{\alpha}(1+\alpha n + \alpha p))$$

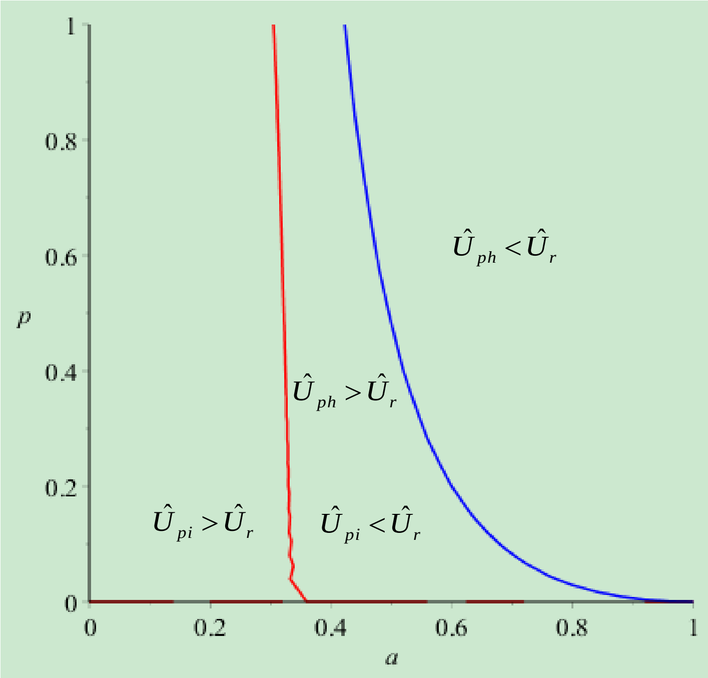
$$\begin{aligned}(1-\alpha)(\hat{U}_{ph} - \hat{U}_r) &= (1-\alpha)(\ln((1-\alpha)A - (1+p)\hat{k}_p^{1-\alpha}) + 2\alpha\ln(\hat{k}_p) + \ln(1+\alpha p) - \ln((1-\alpha)A - \hat{k}_r^{1-\alpha}) - 2\alpha\ln(\hat{k}_r) - \ln\alpha) \\ &= \ln(\frac{(\frac{4\alpha p - 2p + 2}{1+p})^{2\alpha}(\frac{2}{\alpha})^{1-\alpha}(\alpha p + 1)^{2-2\alpha}}{(3\alpha p - p + 2)^{1+\alpha}}).\end{aligned}$$

Figure 4 shows a welfare comparison which looks notably similar to the comparison of consumption levels of the elderly in Figure 2. In other words, the consumption consequences of being healthy or unhealthy in old age as in Figure 2 drive the welfare result. For either the healthy case or the unhealthy case, workers are better off on the whole with the flexible retirement policy as long as the production process is sufficiently labor intensive. Otherwise, workers are better off with the mandatory retirement policy.

Figure 4: Comparison of Lifetime Utilities across Retirement Policies

Horizontal axis: relative capital intensity of production α

Vertical axis: probability p *of being healthy when elderly*



4. Conclusion

Creating a retirement policy which satisfies a population of citizens facing increasing lifespan is a challenge for governments which are simultaneously fiscally burdened by demographic changes and pay-as-you-go pension programs. In an effort to understand the general equilibrium consequences of a flexible delayed retirement program on both labor and capital markets, as well as the welfare consequences of such a policy, we compare under autarky, the welfare consequence of a flexible retirement program to the current status quo in China, which is the mandatory retirement upon reaching a particular age.

Our model is intuitive and is further able to derive some less obvious insights about the ideal retirement policy. Our results show that even more important than a country's health status of the elderly, the appeal of a flexible retirement policy lies in the production technology that is dominant in the economy. In particular, flexible retirement may be more ideal for labor intensive economies, while mandatory retirement may be ideal for capital intensive economies.

We see several directions for future research. First of all, our study has not addressed any potential pension or social security program of the government which could assist citizens in the challenge of their retirement planning (see for example, Crawford and Lilien, 1981). Designing an optimal pension program under the possibility of flexible retirement is one important direction, including the effects on China's fiscal system (see Huang, Kuang and She, 2012). Secondly, our model does not yet consider heterogeneity in workers besides their health status when old. A simulation based study under the possibility of flexible retirement could help generate more concrete policy insights. Finally, both empirical and theoretical work is needed to design a flexible retirement program which is incentive compatible and induces workers to reveal their health status truthfully.

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