Flexible or Mandatory Retirement? Welfare Implications of Retirement Policies for a Population with Heterogeneous Health Conditions

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Abstract:[1]

A flexible retirement policy has often been proposed as a solution to address the social dilemma of individuals in the population having different desired retirement ages. We analyze such a policy in an overlapping generations general equilibrium framework, where individuals differ in terms of their health condition at the standard retirement age, and therefore in their suitability for remaining in the labor force beyond the standard age. In our model, workers know about their future health condition when they are young, and adjust their savings and labor supply accordingly in order to maximize their lifetime utility. The applies to situations in which workers can fairly accurately predict their health status based upon personal, family, community and professional health status tendencies. We compare the flexible retirement policy to the mandatory retirement policy in the labor and capital markets, and the effects on savings and wages in the aggregate for both healthy and unhealthy, young and elderly cohorts. For economies with sufficiently high capital intensity of production and high levels health among the elderly, a flexible retirement policy can result in a welfare reduction compared to a mandatory retirement policy. Our study links the social desirability of the two retirement policies to the technological and population structure of the economy.

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1. Introduction

As countries around the world struggle to cope with aging populations, some of the most frequently discussed policy issues pertain to the management of retirement programs. While some countries have adopted policies designed towards flexibility of the age of retirement, many countries still maintain a uniform retirement age across the population or are in the process of implementing retirement policy reforms.

In China, where increasing the retirement age is already part of an official plan to reform the government's pension system, an estimated 68% (People's Daily) to 95% (China Youth Daily, Sohu) of survey respondents are opposed to raising the retirement age.[2,3] Currently, the retirement age stands at age 60 for male employees (age 55 for male workers engaging in heavy manual labor), and age 50 for female employees (age 45 for female workers engaging in heavy manual labor). While this retirement age may appear low by the standards of industrialized countries, much of China's labor force engages in high work intensity, including factory work and other manual labor (Lien, Wang and Zheng, 2016). 60 percent of respondents believed that they would be physically incapable of working by the proposed retirement age of 65 (China Youth Daily, Sohu). At the same time, over 73 percent of respondents supported the idea of flexible (early or late) retirement instead of a one-size fits all policy (People's Daily).

Despite the administrative simplicity which comes with imposing a uniform retirement age, members of the older generation may vary substantially in terms of how much longer they would like to work. Some individuals may strongly wish to retire early for health or financial reasons, while other individuals for analogous reasons, may prefer to prolong their participation in the labor force. The differing desires of elderly citizens regarding the age at which they actually prefer to retire highlights the heterogeneous circumstances of elderly workers. Indeed, in the above surveys conducted by news outlets (People's Daily; China Youth Daily, Sohu) find a relatively high level of support for flexible retirement policies instead of policies which are one-size-fits-all.

It is well-recognized that China is one of several countries in the world facing an aging population, which poses a serious challenge to their retirement policy and pension system (World Bank, 1994; James 2001; Dorfman, Holzmann, O'Keefe, Wang, Sin and Hinz, 2013). After the development of a market-oriented economy, a large scale social insurance program was needed to replace the employer-based pension model (Song and Chu, 1997; Tang and Ngan, 2001). Reform of the elderly insurance system and personnel reform led to a system which imposed an official mandatory retirement age on workers across industries and personnel levels (Li and Zhou, 2005).

The health status of older workers in China is an issue of direct relevance to the retirement scheme

[2] "Nearly 70 pct against retirement age increasing", English.news.cn, Xinhua, Nov. 1, 2013, Ed: Hou Qiang; The survey is of 1062 people in 11 cities across China, using the online survey on people.com.cn (operated by People's Daily), 96.5 of whom are covered by China's old age pension program. 68.6 percent of respondents were against raising the retirement age, and 73.5 percent of respondents supported flexible retirement. "The top three measures that the respondents' (sic) thought should be implemented in reforming the pension system were adopting a flexible retirement mechanism instead of a one-size-fits-all policy, setting up a multi-pronged system and narrowing the gap of how much pension different groups of employees should receive."

[3] "Most opposed to increasing retirement age", Ecns.cn, China Daily, Sept. 2, 2013, Ed: Wang Fan; The survey is comprised of about 25,300 polled netizens in a survey conducted jointly by Beijing-based China Youth Daily and Sohu. "Nearly 95 percent of some 25,300 polled netizens said they were against the prospect of the retirement age being increased….about 91 percent of respondents said that they were unwilling to work until 65. Most of the surveyed were aged between 24 and 53….some 60 percent believed they would be physically incapable of working up to 65….More than 60 percent of those polled believed China should introduce more flexible retirement arrangements for people from different walks of life."

problem and proposed solutions. For example, migrant workers, who comprise an important part of the modern working economy often have very favorable health status when young, but may be exposed to negative health experiences later in life through adverse working conditions (Hesketh, Ye, Li and Wang, 2008). Wang et al (2012) find in a survey of retirees in China, that between 3% to 15% of respondents were exposed to various sources of hazardous working conditions over the course of their lifetime labor market participation.

The heterogeneity in health status of older citizens of China has been examined in several studies. Strauss et al (2010) find among the elderly in China, unmeasured community influences on health are notably important compared to the elderly in other countries, suggesting that differences in health status can be community-specific. In addition, air pollution in China's cities are found to significantly affect the health of the elderly (Sun and Gu, 2008), creating heterogenous health conditions on a national level. Feng, Wang and Jones (2013) find substantial heterogeneity in the health of the elderly based on province of residence and family support provided. Gender is another potential source of heterogeneity among retirees in China. Gu, Dupre, Warner and Zeng (2009) find that although women worldwide are typically found to live longer than men and under worse health conditions, over the period 1992 to 2002, the health status of women in China increased more than that of men (see also Yu and Sarri, 1997). Finally, the health and financial status of the elderly has been documented to substantially affect the life choices of family members such as in migration decisions (Giles and Mu, 2007; see also Wang, Li and Lien, 2016 and Martin, 1988). For these and other fundamental reasons, analyzing dynamic decisions of a health-heterogeneous population under different retirement schemes is important for the case of China.

We analyze the general equilibrium result and welfare consequence of having a mandatory versus a flexible retirement policy in which workers who are healthy can continue to work in the labor market, while those who are unhealthy retire at the standard retirement age. The model utilizes an overlapping generations framework, comprised of workers who know their eventual elderly health status when they are young. This scenario corresponds to situations where pre-retirement workers have an idea of their future health status through indicators such as their current health status, family health history, or health status of retired aged workers in their profession or regional community. While such an assumption may appear strong, the tendency for individuals to use their private information about health status to make decisions about the future is well documented in the insurance literature on adverse selection (see Cohen and Siegelman, 2010 for a survey). Indeed, Zhang and Wang (2008) found evidence of adverse selection in the community-based health insurance program of rural China. Another way of viewing this assumption underlying our analysis is that workers, even if facing a challenge in predicting their exact future health status, may find it feasible to assess how likely they are to be able to work into their old age. The model enables us to examine the impacts of a flexible retirement policy on labor markets and capital markets in the long run equilibrium. To our knowledge, other studies have not examined these potential retirement policies using this overlapping generations general equilibrium approach.

The basic equilibrium results of the model are that capital accumulation, wages and savings are lower, while labor supply and interest rates are higher, under the flexible retirement policy compared to the mandatory retirement policy. Relevant to modern technological developments in economies, we find that the welfare consequences of each policy depend on the technology and health status parameters of the economy. In particular, when the capital intensity of production is high, and the social health level is high (given by the proportion of workers who are still healthy when old), standard measures of social welfare such as workers' wealth, consumption and utility can be lower under the flexible retirement policy than the mandatory policy. This is surprising given that the result holds across worker health status types and across either stage of workers' lives. The main intuition is that in a flexible retirement setting, the aggregate incentive to accumulate capital is reduced due to a high fraction of workers being able to participate in the labor market late in life. When combined with a production economy that relies heavily on capital, the welfare of workers in the aggregate is diminished. On the other hand, when both capital intensity of production and social health status are low, these welfare measures can indeed be higher under the flexible retirement policy compared to the mandatory one, due to the reverse reasoning. The results suggest that societies may proceed cautiously when implementing flexible retirement policies for highly healthy and capital-intensive economies.

Although our interest in the two retirement policies may be inspired by China's retirement scenario, our model and analysis are fully general and can apply to any country's economy. Therefore, our study contributes to the theoretical literature on analyzing retirement policies. Mandatory retirement policies based upon the age of workers have sometimes been considered controversial on the basis of age discrimination and potentially less than optimal from a social welfare standpoint (see Palmore, 1972; Gunderson, 1998). Lazear (1979) proposes that mandatory retirement policies can be understood as an efficient solution to a dynamic agency problem over workers' lifetimes. At the same time, flexible retirement policies have also been theoretically analyzed, particularly from the perspective of pension policies. Simonovits (2003) uses a mechanism design approach to find the optimal linear pension rules under a flexible retirement scheme, while Eso and Simonovits (2003) finds the optimal non-linear rule. Flexible retirement is also analyzed in van Vuuren (2014), which discusses the necessary conditions for flexible retirement to serve as an insurance against pension risk.

The empirical literature on the relationship between health status and retirement choices raises several key findings, many of which focus on whether poor health is a driving force behind retirement decisions. Quinn (1977) finds that health status and Social Security eligibility are the greatest predictors of early retirement, with significant interaction effect between the two factors. Dwyer and Mitchell (1999) find that in the United States, health problems are a greater influence on retirement plans than economic concerns, which is contrary to some prior findings in the literature (Bazzoli, 1985). McGarry (2004) also finds that health status weighs heavily in retirement decisions, and that the reason is not the 'justification bias' of retirees trying to provide reasons for their retirement decision ex-post (Anderson and Burkhauser, 1985). Peng, Liu, Chen and Chan (2016) using survey data from Taiwan, find that older workers with poor health have higher utility of work and consumption over leisure compared to their healthier counterparts, and thus early retirement behavior may be driven by lower wages and life expectancy rather than an intrinsic preference for retirement. Another line of literature examines the reverse causal relationship, whether retirement increases health status. Ekerdt, Bosse, and LoCastro (1983) find that the claims of increased health after retirement are most likely among individuals with previously strenuous job demands but may not be well-substantiated overall. Using a regression discontinuity approach, Johnston and Lee (2009) find that retirement increases overall well-being and mental health, although not necessarily so for physical health.

The paper proceeds as follows: Section 2 describes the model set up and solves the steady state equilibrium under different policies; Section 3 conducts welfare analysis and provides policy implications; Section 4 concludes and discusses future research directions.

2. Model

2.1 Basic Setup

Similar to the framework in Feng, Lien and Zheng (2017), we consider an infinite horizon two-period overlapping generations model under a production economy. Overlapping generations framework is frequently used to model the interactions between generations in a population. A recent example is Liu and Liu (2017) which uses this framework to model the decision of Chinese households to bear another child under China's two-child policy.

Agents have two stages of life: a young stage in which they are healthy and will participate in the labor market for sure; and an elderly stage in which their employment outcome depend both on retirement policies and their health conditions. Under the mandatory retirement policy, the agent must retire and is not allowed to work during the elderly stage regardless their health condition. Under the flexible retirement policy, the agent can work if he is healthy, and cannot work if he is unhealthy, where different individuals may have different health conditions. The main difference between this paper and Feng, Lien and Zheng (2017) is that we consider population with heterogeneous but certain health conditions while Feng, Lien and Zheng (2017) study ex-ante homogeneous population with uncertain health condition.

Specifically, we consider a situation where individuals when young can predict their health condition when old when making saving and consumption decisions in their youth. Although young workers are assumed to know their future health status with certainty, the population as a whole maintains a proportion $p \in (0,1]$ of healthy individuals in the labor force. The parameter p can be used to measure the level of population health condition.

Each agent has a per-period utility function $u(c) = \ln c$, where $c > 0$ and a time discount factor $\beta \in (0,1]$. Note that utility depends only on consumption and not on leisure, so that the agent will spend all his time working, if possible. The agent will make a consumption and savings decision in his youth, which will at least partially determine the amount of money available to him in the elderly stage. The agent's goal is to maximize the sum of their utilities over the two stages of life, young and old.

Production in the economy is given by the Cobb-Douglas production function, with constant returns to scale $Y = F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$, where L denotes labor, K denotes capital, A denotes technological progress, and α denotes the level of capital intensity of production. The savings of agents are converted to capital at unit conversion in their stage of youth, and are returned to agents in the elderly stage along with a return on the capital investment. From the firm's perspective, there is a one period delay in receiving its capital stock, over which it depreciates by discount factor δ.[4] The population in period t is denoted by N_t where the population evolves according to $N_{t+1} = (1+n)N_t$. Note that we focus on the case of autarky, so there is no mandatory savings or pension program in place.

[4] As we will show later in Section 2.2.2 "The Firm's Problem", the value of δ alone has no effect on equilibrium allocation and welfare consequences except that the equilibrium capital interest rate is negatively correlated with δ. What really matters is the sum of the capital interest rate and the capital depreciation rate, which determines the gross rate of return on capital. For simplicity, we can set $\delta = 0$, which means no depreciation in production for each period, or $\delta = 1$, which means full depreciation in production for each period.

We study the general equilibrium results of the economy under two alternative retirement policies: the Mandatory retirement policy and the Flexible retirement policy. Under the Mandatory retirement policy all workers regardless their health conditions must retire when old, while under the Flexible retirement policy one's employment status depends on his or her health condition. The notation throughout the paper is that 'M' denotes the Mandatory Retirement Policy and 'F' denotes the Flexible Retirement Policy.

2.2 Mandatory Retirement Policy

We first examine the workers' and firm's problem under the policy of mandatory retirement. In this case, when workers reach the elderly stage, they retire regardless of their health status. This reflects the current policy in China and some other countries. The analysis for this policy environment is simple and it is technologically equivalent to the standard overlapping generations economy with production.

The Mandatory Retirement Policy can also be understood as a special limit case of the Flexible Retirement Policy where the level of population health condition approaches zero ($p=0$, namely, all workers are unhealthy when they are old).

2.2.1 The Workers' Problem

Generation-t's problem is to maximize their lifetime utility:

$$Max_{c_t^t, c_{t+1}^t} \ \ln c_t^t + \beta \ln c_{t+1}^t$$

$$\begin{aligned} s.t.\ & c_t^t = w_t - s_t; \\ & c_{t+1}^t = R_{t+1} s_t; \\ & c_t^t, c_{t+1}^t \geq 0. \end{aligned}$$

The first order condition for the worker is given by:

$$-\frac{1}{w_t - s_t} + \frac{\beta}{s_t} = 0$$

From this we obtain the optimal saving decision $s_t^* = \dfrac{\beta w_t}{1+\beta}$. From the condition that consumption in either stage of life must be positive $c_t^t, c_{t+1}^t > 0$, we obtain the range of savings $s_t^* \in (0, w_t)$.

2.2.2 The Firms' Problem

The representative firm's problem is given by the profit maximization problem:

$$Max_{k_t} \ Ak_t^{\alpha} - w_t - r_t k_t - \delta k_t ,$$

with $y = f(k) = Ak_t^{\alpha} = \dfrac{F(K_t, L_t)}{L_t}$ and $\alpha A k_t^{\alpha-1} = r_t + \delta = R_{t+1}$, where $k_t \equiv \dfrac{K_t}{L_t}$ represents the capital per labor. [5]

Since the firm's production is constant returns to scale, there will be zero profit such that the firm's output is equal to its wage bill and return on capital paid, that is, $Ak_t^{\alpha} - k_t R_{t+1} = w_t$.

The expression for the capital stock is simplified by the certainty faced by workers in the early stage of life, such that $k_{t+1} = \frac{s_t^*}{1+n}$. Since $s_t^* = \frac{\beta w_t}{1+\beta} = \frac{\beta}{1+\beta}(Ak_t^{\alpha} - k_t R_{t+1}) = \frac{\beta}{1+\beta}(Ak_t^{\alpha} - k_t \alpha A k_t^{\alpha-1})$, we can express k_{t+1} in terms of k_t, that is, $k_{t+1} = \frac{A\beta(1-\alpha)k_t^{\alpha}}{(1+n)(1+\beta)}$.

The above condition yields the steady state solutions $\hat{k}_1 = 0, \hat{k}_2 = (\frac{A\beta(1-\alpha)}{(1+n)(1+\beta)})^{\frac{1}{1-\alpha}}$ of which, $\left.\frac{dk_{t+1}}{dk_t}\right|_{\hat{k}_1=0} = +\infty > 1$ is not stable, and $\left.\frac{dk_{t+1}}{dk_t}\right|_{\hat{k}_2} = \alpha < 1$ is stable.

To summarize, under the mandatory retirement policy, in the steady state, the capital per labor is $\hat{k}_M = (\frac{A\beta(1-\alpha)}{(1+n)(1+\beta)})^{\frac{1}{1-\alpha}}$, the individual saving of a worker regardless his health condition is $\hat{s}_M = (1+n)\hat{k}_M$, the wage is $\hat{w}_M = A(1-\alpha)(\hat{k}_M)^{\alpha}$, the gross interest rate is $\hat{R}_M = \alpha A(\hat{k}_M)^{\alpha-1}$, the individual consumption of a young worker is $\hat{c}_{y,M} = A(1-\alpha)(\hat{k}_M)^{\alpha} - (1+n)\hat{k}_M$, the individual consumption of an old worker regardless of his health condition is $\hat{c}_{o,M} = A\alpha(1+n)(\hat{k}_M)^{\alpha}$.

2.3 Flexible Retirement Policy

For the flexible retirement policy environment, the analysis will be relatively more complicated due to the heterogeneous health condition assumption. For the consumers' problem, we will first analyze the utility maximization problem for the healthy workers, and then analyze the problem where the worker is unhealthy when old.

2.3.1 The Workers' Problem

Healthy Worker's Problem:

If a worker knows he will be healthy when old, his utility maximization problem is given by

$$\max u_h = \ln c_t^t + \beta \ln c_{t+1}^t$$

[5] Note that $r_t + \delta = R_{t+1}$ simply means that the return of capital plus the depreciation of capital from the firm side should be equal to the return on savings from the consumer side. Since R_{t+1} instead of r_t is our main focus, the value of δ has no effect on the equilibrium allocation.

$$s.t.\ c_t^t = w_t - s_t,$$
$$c_{t+1}^t = w_{t+1} + R_{t+1}s_t,$$
$$c_t^t, c_{t+1}^t \geq 0.$$

From the first order condition, we have

$$-\frac{1}{w_t - s_t} + \frac{\beta R_{t+1}}{R_{t+1}s_t + w_{t+1}} = 0 .$$

Solving for the saving term, we obtain $s_t^* = \frac{\beta R_{t+1} w_t - w_{t+1}}{R_{t+1}(1+\beta)}$. The strictly positive consumption condition in both stages of life $c_t^t, c_{t+1}^t > 0$ requires that the range of savings satisfies $s_t^* \in (-\frac{w_{t+1}}{R_{t+1}}, w_t)$.

Unhealthy Worker's Problem:

If the worker will be unhealthy so that he will not work when he is old his utility maximization problem is given by

$$\max u_i = \ln c_t^t + \beta \ln c_{t+1}^t$$

$$s.t.\ c_t^t = w_t - s_t,$$
$$c_{t+1}^t = R_{t+1}s_t,$$
$$c_t^t, c_{t+1}^t \geq 0.$$

Note that the unhealthy worker's problem described above is identical to the worker's problem under the mandatory retirement policy since in both scenarios the unhealthy worker works when young and retires when old. Based on the previous analysis, the optimal saving decision is $s_t^* = \frac{\beta w_t}{1+\beta}$, $s_t^* \in (0, w_t)$.

Total savings in the entire economy is derived by aggregating the savings across the heterogeneous population, that is, $p\frac{\beta R_{t+1} w_t - w_{t+1}}{R_{t+1}(1+\beta)} + (1-p)\frac{\beta w_t}{1+\beta}$.

2.3.2 The Firms' Problem

The firm's problem $Max_{k_t}\ Ak_t^\alpha - w_t - r_t k_t - \delta k_t$ is similar to that in the Mandatory Retirement Policy in obtaining the zero profit conditions, namely $\alpha A k_t^{\alpha-1} = R_t$, $Ak_t^\alpha - k_t R_t = A(1-\alpha)k_t^\alpha = w_t$.

From the law of motion for per capita capital, we have

$$k_{t+1} = \frac{p\frac{\beta R_{t+1}w_t - w_{t+1}}{R_{t+1}(1+\beta)} + (1-p)\frac{\beta w_t}{1+\beta}}{1+n+p} = \frac{\beta R_{t+1}w_t - pw_{t+1}}{R_{t+1}(1+n+p)(1+\beta)} \geq 0 .$$[6]

Substituting R_{t+1}, w_{t+1}, w_t with respect to k_t, and letting $k_{t+1} = k_t$, we solve for the capita capital stock in the steady state:

$$\hat{k}_1 = 0\,;\;\; \hat{k}_2 = \left(\frac{A\alpha\beta(1-\alpha)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p}\right)^{\frac{1}{1-\alpha}} > 0 .$$

From $k_{t+1} = \frac{A\alpha\beta(1-\alpha)k_t^{\alpha}}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p}$, we have $\frac{dk_{t+1}}{dk_t} = \frac{(1-\alpha)\alpha^2\beta A k_t^{\alpha-1}}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p}$.

$\left.\frac{dk_{t+1}}{dk_t}\right|_{\hat{k}_1=0} = +\infty > 1$, so $\hat{k}_1$ is not stable; $\left.\frac{dk_{t+1}}{dk_t}\right|_{\hat{k}_2} = \alpha < 1$ so $\hat{k}_2$ is stable.

Therefore, we have $\hat{k}_F = \left(\frac{A\alpha\beta(1-\alpha)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p}\right)^{\frac{1}{1-\alpha}}$.

To summarize, under the flexible retirement policy, in the steady state, the capital per labor is $\hat{k}_F = \left(\frac{A\alpha\beta(1-\alpha)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p}\right)^{\frac{1}{1-\alpha}}$, the individual saving of a healthy worker is $\hat{s}_{F_h} = \frac{\beta A(1-\alpha)}{1+\beta}(\hat{k}_F)^{\alpha} - \frac{(1-\alpha)}{(1+\beta)\alpha}\hat{k}_F$, the individual saving of an unhealthy worker is $\hat{s}_{F_u} = \frac{\beta A(1-\alpha)}{1+\beta}(\hat{k}_F)^{\alpha}$, the wage is $\hat{w}_F = A(1-\alpha)(\hat{k}_F)^{\alpha}$, the gross interest rate is $\hat{R}_F = \alpha A(\hat{k}_F)^{\alpha-1}$, the individual consumption of a healthy young worker is $\hat{c}_{y,F_h} = \hat{w}_F - \hat{s}_{F_h} = A\hat{k}_F^{\alpha}(1-\alpha) - \frac{(\beta\alpha A\hat{k}_F^{\alpha-1} - 1)\hat{k}_F(1-\alpha)}{\alpha(1+\beta)}$, the individual consumption of an unhealthy young worker is $\hat{c}_{y,F_u} = \hat{w}_F - \hat{s}_{F_u} = \hat{w}_F\frac{1}{1+\beta} = A\hat{k}_F^{\alpha}\frac{1-\alpha}{1+\beta}$, the individual consumption of a healthy older worker is $\hat{c}_{o,F_h} = A\hat{k}_F^{\alpha}(1-\alpha) + \alpha A\hat{k}_F^{\alpha-1}\frac{(\beta\alpha A\hat{k}_F^{\alpha-1} - 1)\hat{k}_F(1-\alpha)}{\alpha(1+\beta)}$, the individual consumption of an unhealthy older worker is $\hat{c}_{o,F_u} = \alpha A\hat{k}_F^{\alpha-1}\frac{\beta}{\beta+1}A(1-\alpha)\hat{k}_F^{\alpha}$.

As a special case of interest, when $\beta=1$ and $n=0$, we have $\hat{k}_F = \left(\frac{A\alpha(1-\alpha)}{\alpha p + 2\alpha + p}\right)^{\frac{1}{1-\alpha}}$.

[6] Note that under the flexible retirement policy, even though the savings of the healthy workers could be potentially negative by borrowing from the firm, the overall saving from both the healthy and unhealthy workers are always positive because in equilibrium the capital accumulation is always non-negative.

Furthermore, we could get $\hat{R}_F = \alpha A\hat{k}^{\alpha-1} = \dfrac{\alpha p + 2\alpha + p}{1-\alpha}$; $\hat{w}_F = A\hat{k}^{\alpha}(1-\alpha) = A(1-\alpha)(\dfrac{A\alpha(1-\alpha)}{\alpha p + 2\alpha + p})^{\frac{\alpha}{1-\alpha}}$.

3. Comparison across Retirement Policies

In this section, we first compare the equilibrium results of the economy and then the welfare consequences for the consumers, under the mandatory retirement policy and the flexible retirement policy. The equilibrium results refer to the economic variables including capital, labor, interest, wage, and saving. The welfare consequences for the consumers refer to the consumers' income (or wealth in the aggregate), consumption, and utility (or welfare in the aggregate), at different stages of life (young versus old), for different types (healthy versus unhealthy), from the lifetime perspective (lifetime income, lifetime consumption, and lifetime utility), and from the social perspective (social wealth, social consumption, and social welfare).

For the comparison on the equilibrium results of the economy, we stick to the general setup to allow $n \geq 0$ and $0 < \beta \leq 1$. For the comparison of welfare consequences for consumers, since the theoretical analysis comparing the mandatory retirement policy and the flexible retirement policy can be quite complex, and in many cases a tractable condition and closed-form solution is not available, we use numerical simulations to pin down the relevant cutoff ranges.

3.1 Comparison of Equilibrium Results

Based on the derived equilibrium outcomes from the previous section, we can compare the equilibrium consequences of five basic economic variables under the mandatory retirement policy and the flexible retirement policy. These five economic variables are **Labor**, **Capital**, **Wage** (price of labor), **Interest Rate** (price of capital), and **Saving**.

3.1.1 Labor Supply

Note that in our model labor supply is assumed to be inelastic for simplicity, and leisure does not enter the utility function. Therefore, the difference in labor supply between these two policy environments depends only on whether the healthy old people are allowed to continue working or not. It is easy to know that $\dfrac{\hat{l}_F}{\hat{l}_M} = \dfrac{1+n+p}{1+n} > 1$.

Proposition 0 (Labor): Labor supply is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

This insight follows straightforwardly from the labor market activity permitted by the flexible retirement policy compared to the mandatory one. Since a fraction of workers in the population are healthy when old and can supply labor if permitted to, the aggregate labor supply will be higher under flexible retirement than under the scenario where all workers are forced to retire in old age.

3.1.2 Capital Accumulation

From the analysis in Section 2, we know the equilibrium capital per labor accumulation level under both retirement policies: $\hat{k}_M = (\frac{A\beta(1-\alpha)}{(1+n)(1+\beta)})^{\frac{1}{1-\alpha}}$ and $\hat{k}_F = (\frac{A\alpha\beta(1-\alpha)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p})^{\frac{1}{1-\alpha}}$.

Thus, we have $\frac{\hat{k}_F}{\hat{k}_M} = (\frac{(1+\beta)\alpha(1+n)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p})^{\frac{1}{1-\alpha}} = (\frac{(1+\beta)\alpha(1+n)}{(1+\beta)\alpha(1+n) + p(\beta\alpha+1)})^{\frac{1}{1-\alpha}}$, where the difference between numerator and denominator is $-p(\beta\alpha+1)$, therefore we have $\frac{\hat{k}_F}{\hat{k}_M} < 1$.

Proposition 1 (Capital): The equilibrium capital accumulation is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Proposition 1 asserts that a mandatory retirement policy, when compared to a flexible policy, leads to a larger capital accumulation in the economy. When workers are required to retire by the standard retirement age, they need to save more when they are in the labor force in order to accumulate enough savings for the retirement period. On the other hand, when workers who are healthy are permitted to continue working past the standard retirement age, the need for savings in old age is reduced, resulting in lower capital accumulation in the aggregate.

3.1.3 Wage

Based on the result from Proposition 1, we can easily derive the comparison result for equilibrium wage under different retirement policies.

Corollary 1 (Wage): The equilibrium wage is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Corollary 1 states that in the steady state equilibrium, we can expect wages to be lower under the flexible retirement policy than under mandatory retirement. This result is intuitive since under flexible retirement, we have a greater aggregate labor supply which lowers workers' marginal product and equilibrium wages in the labor market.

3.1.4 Interest Rate

Similarly, based on the result from Proposition 1, we can easily derive the comparison result for equilibrium interest rate under different retirement policies.

Corollary 2 (Interest Rate): The equilibrium interest rate is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Corollary 2 states that the interest rate in the capital market under the flexible retirement policy is greater than that in the mandatory retirement policy. The intuition is that when all workers are forced to retire at the standard retirement age, there is greater saving occurring in due to the restriction of labor market participation when elderly. In other words, young workers must save for their retirement since they will be restricted from earning wages when old due to mandatory retirement. These greater savings levels in the macroeconomy lead to lower interest rates in the capital market.

3.1.5 Saving

We are also interested in comparing the equilibrium total savings under different retirement policies. Based on the equilibrium analysis in Section 2, we know

$$\frac{\hat{s}_F}{\hat{s}_M}=\frac{1+n+p}{1+n}\frac{\hat{k}_F}{\hat{k}_M}=(\frac{1+n+p}{1+n})^{\frac{-\alpha}{1-\alpha}}(\frac{1+n+p}{1+n}\frac{(1+\beta)\alpha(1+n)}{\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p})^{\frac{1}{1-\alpha}}.$$

Since $(1+n+p)(1+\beta)\alpha-(\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p)=p(\alpha-1)<0$, we know that the following inequality hold:

$$(\frac{(1+n+p)(1+\beta)\alpha}{\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p})^{\frac{1}{1-\alpha}}<1.$$

The above condition combining with the fact $(\frac{1+n+p}{1+n})^{\frac{-\alpha}{1-\alpha}}<1$, implies that $\frac{\hat{s}_F}{\hat{s}_M}<1$.

Proposition 2 (Saving): The equilibrium saving is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Proposition 2 provides additional insight into the finding of Proposition 1, pinpointing the source of higher capital accumulation under the mandatory retirement case to young workers. Since in our model, workers make their key savings decisions when they are young, it naturally follows that the savings behavior of the young drives this effect.

Note that our measures of capital and saving are slightly different because the capital is measured on a per labor basis, and the savings variable measures the aggregate per capita levels. Based on Proposition 1 and Proposition 2, we can infer that capital and savings no matter whether measured as per labor or per capita, are lower under the flexible retirement policy than under the mandatory retirement policy. Since healthy people can work to earn income when old under flexible retirement, they have less incentive to save their wages when they are young, leading to a relatively low level of capital accumulation in equilibrium.

3.2 Comparison of Welfare Consequences

We now conduct welfare analysis under the mandatory retirement policy and the flexible retirement policy. Since in our model the representative firm exhibits a constant-return-to-scale production technology, the firm's equilibrium profit is zero and thus there is no welfare difference from the producers' side under any retirement policy. Therefore, we focus on the welfare consequences for the workers or consumers, under the mandatory retirement policy and the flexible retirement policy. The welfare consequences for consumers refer to the consumers' income (or wealth in the aggregate), consumption, and utility (or welfare in the aggregate).

Individuals with different health conditions (healthy versus unhealthy) may have different welfare outcomes under the different retirement policies. Also, individuals at different stages of their life (young versus old) may have different welfare outcomes under the different retirement policies. Therefore, for each welfare measure (income, consumption, and utility), we first conduct a comparison at the health type-specific and stage-specific individual level (whenever applicable), then at the type-specific individual lifetime level, and finally at the social aggregate level.

For the comparisons under each welfare measure in this subsection, we first investigate the benchmark scenario by assuming that $n=0$ and $\beta=1$, in other words, we do not consider the potential effects of population growth and patience level, respectively. As a robustness check, we then conduct similar analyses on 4 calibration scenarios, each of which represents a different value combination of n and β. Since the parameter values we use are based on real-world Chinese data, the results we derive can be directly relevant for policy implications in China. To be more specific, we consider combinations of high and low values for both discount factor (β) and population growth rate (n).[7] We assume each generation consists of 30 years and yearly data are used to derive the generational data. These parameter values are shown in Table 1.

[7] The low value of yearly discount factor is chosen to be 0.97 while the high value of yearly discount factor is set as 0.99 (Nordhaus 1993; Arrow, 1995; Evans, 2006). The low value of annual population growth rate is 0.5%, calculated as the average growth rate of population in the past 10 years (2008-2017) in China (Data Source: National Bureau of Statistics of China). The high value of annual population growth rate is set as 0.8% to take into account the potential upward sloping trend of population growth rate in China as the decades-old one-child policy was abolished in recent years.

Table 1: Values for Discount Factor (β) and Population Growth Rate (n) for Calibration Scenarios

		Yearly	Generational
β	High value	0.99	0.7397
	Low value	0.97	0.4010
n	High value	0.008	0.2700
	Low value	0.005	0.1614

As the analysis will show, the welfare comparison between these two retirement policies depends on two important parameters: *the level of capital intensity of production* (α) and *the level of health in the population* (p).

3.2.1 Income and Wealth

Type-specific and stage-specific individual level comparison

For young individuals, regardless of their health condition when they are old, their income when they are young is equal to the wage. By Corollary 1, we have $\hat{w}_F < \hat{w}_M$, so a young individual's income is lower under the flexible retirement policy than under the mandatory retirement policy.

For elderly healthy individuals, they can work and receive an income of $\hat{w}_F$ under the flexible retirement policy, while they are not allowed to work and therefore have an income of zero under the mandatory retirement policy. Clearly, an elderly healthy individual's income is higher under the flexible retirement policy than under the mandatory retirement policy.

For elderly unhealthy individuals, they cannot work under either retirement policy and thus their income is the same (equal to zero) under these two retirement policies.

Type-specific individual lifetime level

Based on the above arguments, it is easy to see that for unhealthy individuals, their lifetime income is always lower under the flexible retirement policy than under the mandatory retirement policy, because their income while they are young will be lower under the flexible retirement policy than under the mandatory policy (by Corollary 1, $\hat{w}_F < \hat{w}_M$,) and their income in the elderly stage will be zero under both policies.

However, the comparison is no longer straightforward for healthy individuals, because their income when young is lower under the flexible policy than under the mandatory policy, but their income when elderly is higher under the flexible policy than under the mandatory one. To be more specific, for a healthy individual, his lifetime income is $2\hat{w}_F$ under the flexible retirement policy and $\hat{w}_M$ under the mandatory retirement policy.

Benchmark Scenario ($n=0, \beta=1$):

Since $\hat{w}_F = A(1-\alpha)(\frac{A\alpha(1-\alpha)}{\alpha p + 2\alpha + p})^{\frac{\alpha}{1-\alpha}}$ and $\hat{w}_M = A(1-\alpha)(\frac{A\alpha(1-\alpha)}{2\alpha})^{\frac{\alpha}{1-\alpha}}$ when $n=0, \beta=1$, the ratio of the lifetime income under these two policies is

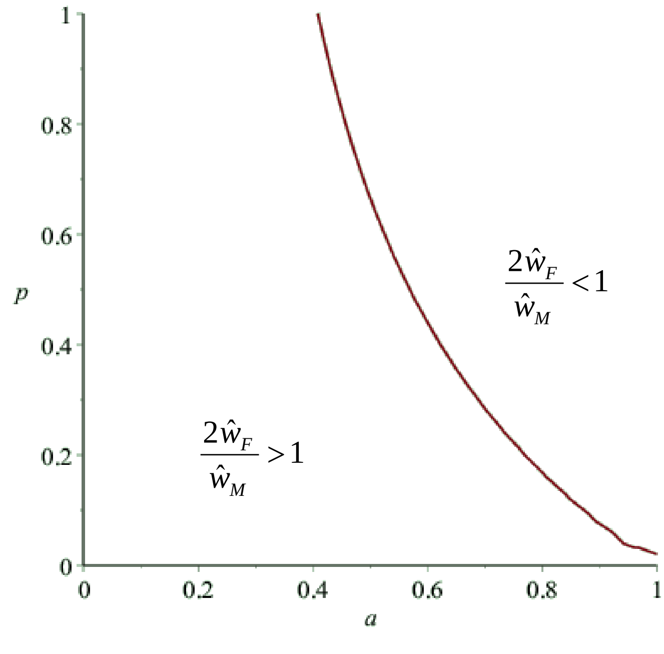
$$\frac{2\hat{w}_F}{\hat{w}_M} = 2(\frac{2\alpha}{\alpha p + 2\alpha + p})^{\frac{\alpha}{1-\alpha}}.$$

The comparison between the value of the above expression and 1 depends on the level of capital intensity of production (α) and the level of health in the population (p). If we put α on the horizontal axis and p on the vertical axis, Figure 1 shows the cutoff condition for the healthy worker's lifetime income comparison between the flexible retirement policy and the mandatory retirement policy.

Figure 1: Ratio of Lifetime Wage Income of the Healthy (Benchmark Scenario)

Horizontal axis: relative capital intensity of production α

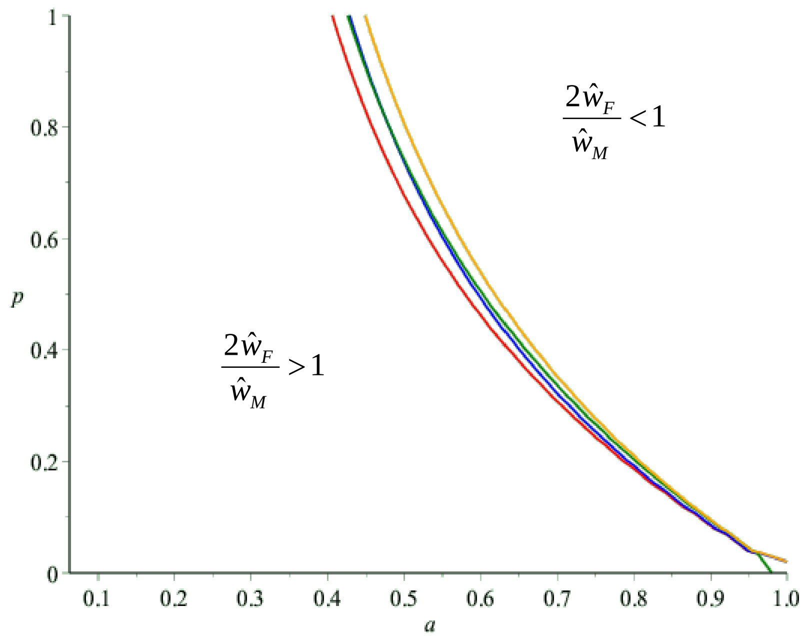
Vertical axis: level of health in the population p



Calibration Scenarios:

Note that in general, we have $\frac{2\hat{w}_F}{\hat{w}_M} = 2(\frac{\alpha(1+n)(1+\beta)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p})^{\frac{\alpha}{1-\alpha}}$. By conducting similar comparisons as in the Benchmark Scenario, we now show the cutoff curves for the healthy worker's lifetime income comparison between the two policies in Figure 1', under each of the 4 Calibration Scenarios, respectively: ① $n = 0.1614, \beta = 0.4010$ (red curve); ② $n = 0.1614, \beta = 0.7397$ (blue curve); ③ $n = 0.2700, \beta = 0.4010$ (green curve); ④ $n = 0.2700, \beta = 0.7397$ (yellow curve).

Figure 1': Ratio of Lifetime Wage Income of the Healthy (Calibration Scenarios)



Comparing Figure 1 and Figure 1', we observe that the cutoff conditions (represented by the curves in the graphs) under different parameter values of n and β display quite a robust general pattern. In addition, an increase of value in n or β tends to shift the cutoff curve to the right, while such a quantitative difference caused by s small change in n or β does not affect the pattern of the cutoff condition qualitatively.

Our analysis yields the following proposition regarding the individual lifetime income comparison.

Proposition 3 (Lifetime Income):

(a) For healthy individuals, the comparison of lifetime income depends on the level of capital intensity of production (α) and the degree of health in the population (p): When both α and p are high, lifetime income is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When both α and p are low, the lifetime income is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

(b) For unhealthy individuals, their lifetime income is always lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Proposition 3 can be understood through the influences of the capital intensity of production parameter and the population health parameter. For high capital intensity of production economies, all else equal, the marginal return to capital is structurally high while the marginal return to labor is lower. These features generally work in the same direction to induce savings from the young, while keeping workers' wages relatively low. The population health parameter represents the fraction of able-to-work elderly workers in any given period, who are simultaneously those individuals with less incentive to save when young. For a larger value of this health parameter, the supply of labor is higher, which further suppresses wages and maintains the higher return to capital.

For those healthy individuals, when these two parameters are high, the wage effect of being able to work when old dominates the lower wage and higher interest rate. The lifetime income is therefore higher for a healthy individual under the flexible retirement policy. For unhealthy individuals, they experience the lower wages associated with the flexible retirement policy when they are young, but are unable to work when they are old to recoup this lost surplus.

3.2.2 Consumption

Type-specific and stage-specific individual level comparison

Healthy individual when young

From the analysis in Section 2, we know $\hat{c}_{y,F_h} = \hat{w}_F - \hat{s}_{F_h} = A\hat{k}_F^{\alpha}(1-\alpha) - \dfrac{(\beta\alpha A\hat{k}_F^{\alpha-1}-1)\hat{k}_F(1-\alpha)}{\alpha(1+\beta)}$,

$$\hat{c}_{y,F_u} = \hat{w}_F - \hat{s}_{F_u} = \hat{w}_F \frac{1}{\beta+1} = A\hat{k}_F^{\alpha} \frac{1-\alpha}{1+\beta}, \text{ and } \hat{c}_{y,M} = \hat{w}_M - \hat{s}_M = A\hat{k}_M^{\alpha}(1-\alpha)-(1+n)\hat{k}_M .$$

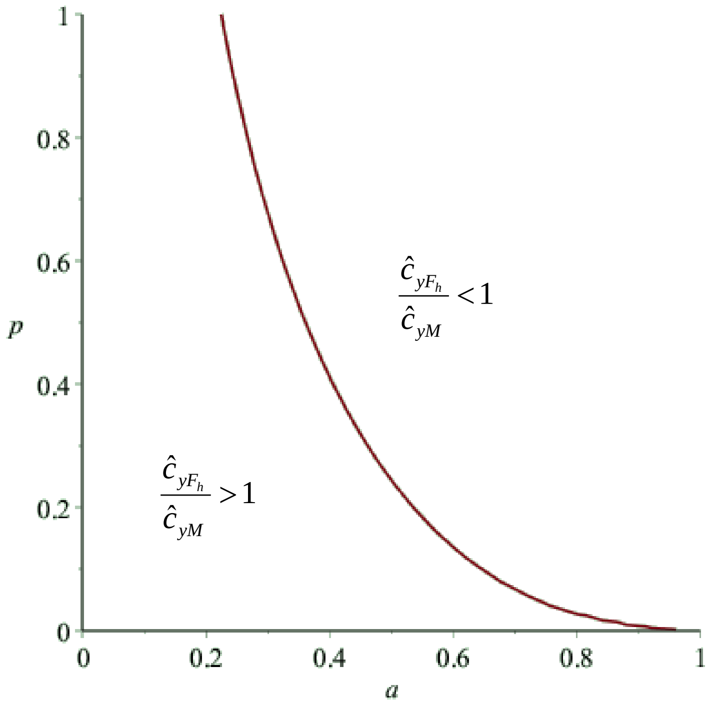
Thus, a young healthy individual will have consumption $\hat{c}_{y,F_h} = A\hat{k}_F^{\alpha}(1-\alpha) - \frac{(\beta\alpha A\hat{k}_F^{\alpha-1}-1)\hat{k}_F(1-\alpha)}{\alpha(1+\beta)}$ under the flexible retirement policy and have consumption $\hat{c}_{y,M} = A\hat{k}_M^{\alpha}(1-\alpha)-(1+n)\hat{k}_M$ under the mandatory retirement policy.

Benchmark Scenario ($n=0, \beta=1$):

When $n=0$ meaning no population growth and $\beta=1$ meaning no discounting between stages of life, the ratio of these two consumption levels is

$$\frac{\hat{c}_{y,F_h}}{\hat{c}_{y,M}} = \frac{(2\alpha)^{\frac{\alpha}{1-\alpha}}(p+1)(\alpha+1)}{((p+2)\alpha+p)^{\frac{1}{1-\alpha}}} .$$

Figure 2: Consumption Comparison for Young Healthy Workers (Benchmark Scenario)



This relationship is illustrated in Figure 2, which plots the level of health in the population (p) against the relative capital intensity of production (α). The solid line represents the cut-off points of these two parameters which separates situations in which consumption of youth is greater under the flexible policy compared to the mandatory policy. The dividing line is nearly vertical when the capital intensity of production is low. There are many cases where for low capital intensities of production, aggregate consumption when young under flexible retirement exceeds that under mandatory retirement, and particularly when a large proportion of individuals are healthy when elderly. This is understandable given that a low capital intensity of production (or high labor intensity), corresponds to higher marginal product of labor and thus higher wages. In such an environment with a flexible retirement policy, especially with a large proportion of healthy individuals, the need to save for retirement is weakened, leading to high consumption levels when young.

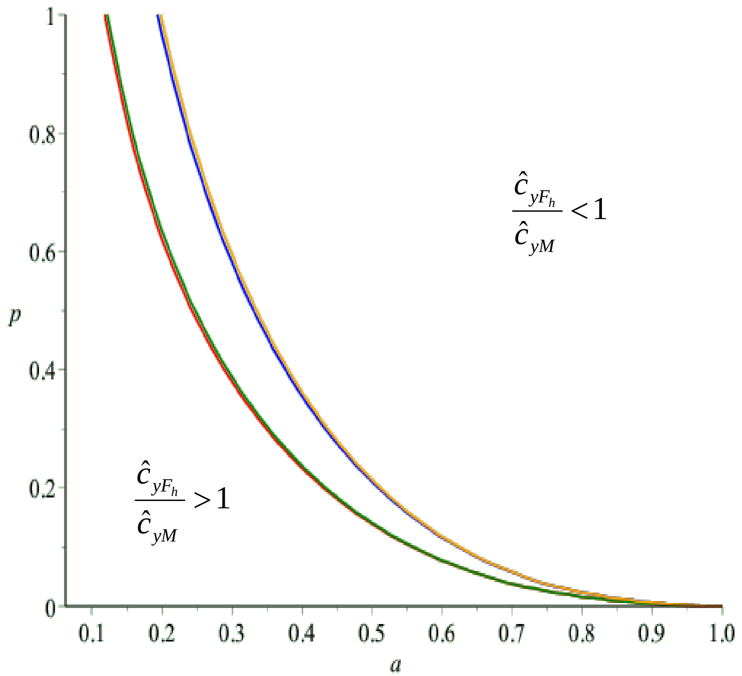
In addition, there is a negative relationship between the two parameters in terms of the cut-off threshold. High capital intensity of production can compensate for low proportion of healthy individuals in maintaining the relationship between consumption when young under the flexible retirement plan and the mandatory retirement plan.

Calibration Scenarios:

Note, $\frac{\hat{c}_{y,F_h}}{\hat{c}_{y,M}} = \frac{\left(\frac{\alpha(1+n)(1+\beta)}{((n+p+1)\beta+n+1)\alpha+p}\right)^{\frac{\alpha}{1-\alpha}}(((n+p)\beta+n+1)\alpha+\beta+p)}{((n+p+1)\beta+n+1)\alpha+p}$.

By conducting similar comparisons as in the Benchmark Scenario, we can show the cutoff curves for the young healthy worker's consumption comparison between the two policies in Figure 2', under each of the four Calibration Scenarios, respectively.

Figure 2': Consumption Comparison for Young Healthy Workers (Calibration Scenarios)



The comparison between Figures 2 and 2' shows that the cutoff curves under different parameter values of n and β also have a robust downward-sloping pattern. In addition, a change in n seems to have little impact on the cutoff condition while an increase in β tends to shift the cutoff curve to the right.

Unhealthy individual when young:

A young unhealthy individual will have consumption $\hat{c}_{y,F_u} = A\hat{k}_F^{\alpha}\frac{1-\alpha}{1+\beta}$ under the flexible retirement policy and have consumption $\hat{c}_{y,M} = A\hat{k}_M^{\alpha}(1-\alpha)-(1+n)\hat{k}_M$ under the mandatory retirement policy.

The comparison of consumption for an unhealthy individual when they are young is analogously derived as:

$$\frac{\hat{c}_{y,F_u}}{\hat{c}_{y,M}} = \left(\frac{\alpha(1+n)(1+\beta)}{\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p}\right)^{\frac{\alpha}{1-\alpha}}.$$

Since $\alpha(1+n)(1+\beta)-(\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p) = -p(\beta\alpha+1) < 0$, we have $\frac{\hat{c}_{y,F_u}}{\hat{c}_{y,M}} < 1$.

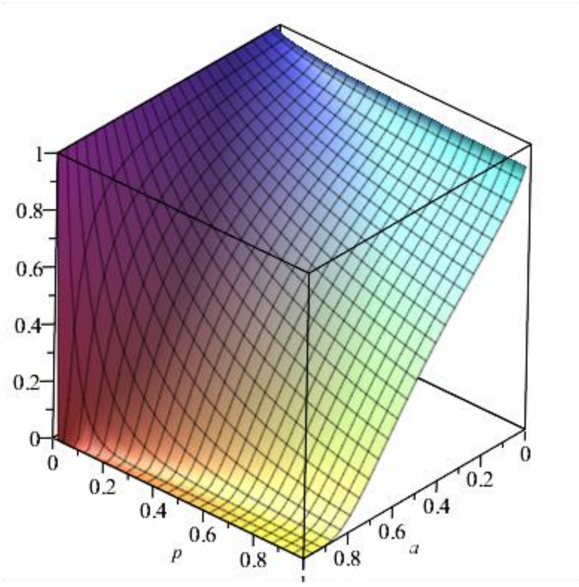
In other words, we have a situation for the unhealthy young workers in which their consumption under the flexible retirement policy is always less than that under the mandatory policy, regardless of the health and technology parameters of the economy.

Although the consumption ratio between flexible and mandatory retirement policies is always less than one for unhealthy workers when they are young, the ratio itself takes different values depending on the population health and technology parameters. This is illustrated in Figure 3, which shows that the flexible to mandatory consumption ratio is predominantly decreasing in the capital intensity of production parameter in the Benchmark Scenario.[8] This is consistent with the intuition of Proposition 3b for unhealthy workers, in that higher capital intensity of production suppresses their lifetime income. Since individuals would generally like to smooth their consumption across the young and elderly stages of life, this lower lifetime income result from a flexible retirement policy carries over to the unhealthy individual's consumption when young. In addition, higher capital intensity leads to higher interest rates, which induces savings among young workers. For an unhealthy worker who

[8] Note that the young unhealthy worker's consumption ratio between flexible and mandatory retirement policies in general depends on 4 parameters: α, p, n, β. We adopt the Benchmark setup ($n=0, \beta=1$) to present the 3-dimensional graph of how such a consumption ratio depends on α and p in Figure 3. Similar patterns can be shown by adopting the different calibration setups.

has no chance to work when old, this strengthens the motive to consume less when young.

Figure 3: Consumption Ratio for Unhealthy Workers (Benchmark Scenario)



These findings lead us to Proposition 4, which states the result for consumption when young for the healthy and unhealthy individuals under the two possible retirement policies.

Proposition 4 (Consumption when Young):

(a) For the healthy, the comparison of consumption when young depends on the level of capital intensity of production (α) and the level of population health condition (p): When both α and p are high, the consumption when young is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When both α and p are low, the consumption when young is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

(b) For the unhealthy, the consumption when young is always lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

To summarize, for an individual who will be unhealthy when elderly, although he cannot work when elderly under either retirement policy, the macroeconomic environment he faces is different under flexible versus mandatory retirement. As mentioned in Corollaries 1 and 2, under flexible retirement, wages are lower and interest rates are higher than for mandatory retirement. Facing lower wages and higher interest rates, the young worker who will be unhealthy in old age has a strong incentive for savings. This will lead to lower consumption while young under the flexible retirement policy compared to the mandatory retirement policy for less healthy workers, regardless of the production technology and proportion of healthy individuals in the population.

Although the ratio of consumption when young from the flexible retirement policy compared to the mandatory policy is always less than 1 regardless of the parameter value, we can also examine how this consumption ratio changes with the population and production parameters of interest. As seen from Figure 3, the main parameter which drives variation in the consumption ratio is the capital intensity of production. As capital intensity of production gets higher, the less favorable consumption levels are in the flexible retirement plan compared to the mandatory plan. This operates through the detrimental effect of capital intensity on the wage of young workers.

We now focus on the comparison of consumption during the elderly stage of life.

Healthy individual when elderly:

Based on the equilibrium analysis in Section 2, the consumption in elderly period for the healthy and unhealthy individuals is given by:

$$\hat{c}_{o,F_h} = \hat{w}_F + \hat{R}_F \hat{s}_{F_h} = A\hat{k}_F^{\alpha}(1-\alpha) + \alpha A\hat{k}_F^{\alpha-1} \frac{(\beta\alpha A\hat{k}_F^{\alpha-1} - 1)\hat{k}_F(1-\alpha)}{\alpha(1+\beta)},$$

$$\hat{c}_{o,F_u} = \hat{R}_F \hat{s}_{F_u} = \alpha A\hat{k}_F^{\alpha-1} \frac{\beta}{\beta+1} A(1-\alpha)\hat{k}_F^{\alpha} = \frac{\beta}{\beta+1}\alpha(1-\alpha)A^2\hat{k}_F^{2\alpha-1},$$

$$\hat{c}_{o,M} = \hat{R}_M \hat{s}_M = \alpha A\hat{k}_M^{\alpha-1}(1+n)\hat{k}_M = \alpha A\hat{k}_M^{\alpha}(1+n).$$

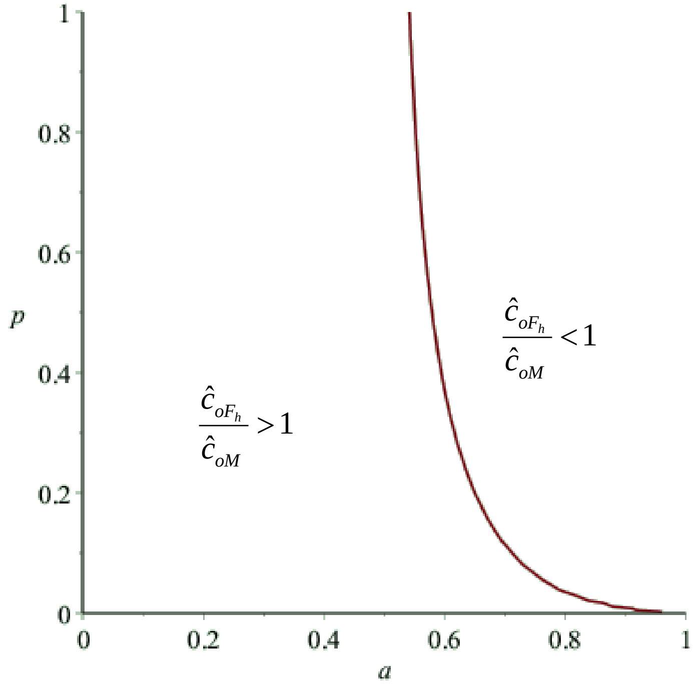
Benchmark Scenario ($n=0, \beta=1$):

When $n=0, \beta=1$, the above results of $\hat{c}_{o,F_h}$ and $\hat{c}_{o,M}$ lead to a consumption ratio of,

$$\frac{\hat{c}_{o,F_h}}{\hat{c}_{o,M}} = \frac{1}{4} \frac{2^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{(p+2)\alpha + p}\right)^{\frac{\alpha}{1-\alpha}} (p+1)(\alpha+1)}{\alpha}.$$

Figure 4 shows the cut-off values for the consumption comparison between the retirement policies for the healthy elderly workers under the Benchmark Scenario. The result looks much like the case for the young healthy workers. Lower capital intensity of production drives higher consumption under the flexible retirement scheme. For especially low proportions of healthy individuals in the population, a high capital intensity of production can also yield higher consumption under flexible retirement.

Figure 4: Consumption Comparison for Elderly Healthy Workers (Benchmark Scenario)



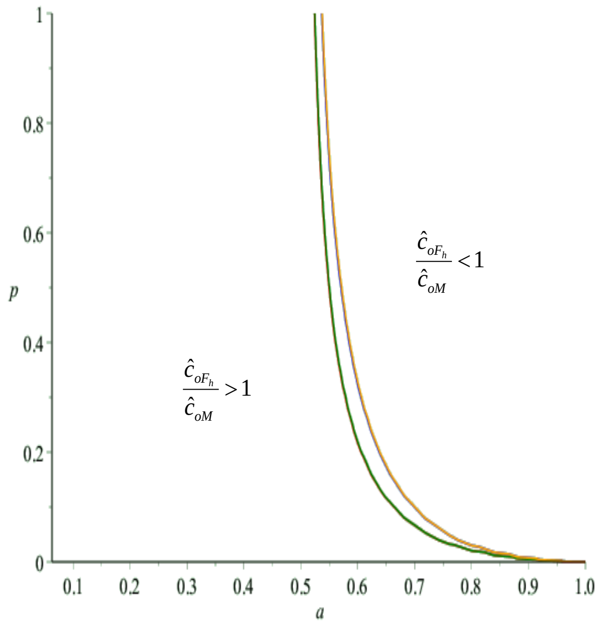
Calibration Scenarios:

Note $$\frac{\hat{c}_{o,F_h}}{\hat{c}_{o,M}} = \frac{\left(\frac{\alpha(1+n)(1+\beta)}{((n+p+1)\beta+1+n)\alpha+p}\right)^{\frac{\alpha}{1-\alpha}}(((n+p)\beta+1+n)\alpha+\beta+p)}{\alpha(1+n)(1+\beta)}.$$

By conducting similar comparisons as in the Benchmark Scenario, we can show the cutoff curves for the elderly healthy worker's consumption comparison between the two policies in Figure 4', under each of the four Calibration Scenarios, respectively.

The comparison between Figures 4 and 4' shows that the cutoff curves under different parameter values of n and β again have a robust downward-sloping pattern. Similar to the young healthy worker's consumption comparison, a change in n seems to have little impact on the cutoff condition for the elderly healthy worker's consumption comparison while an increase in β tends to shift the cutoff curve to the right.

Figure 4’: Consumption Comparison for Elderly Healthy Workers (Calibration Scenarios)



Unhealthy individual when elderly:

The similar analysis can be done for the elderly unhealthy individuals.

The comparison of consumption in elderly period is:

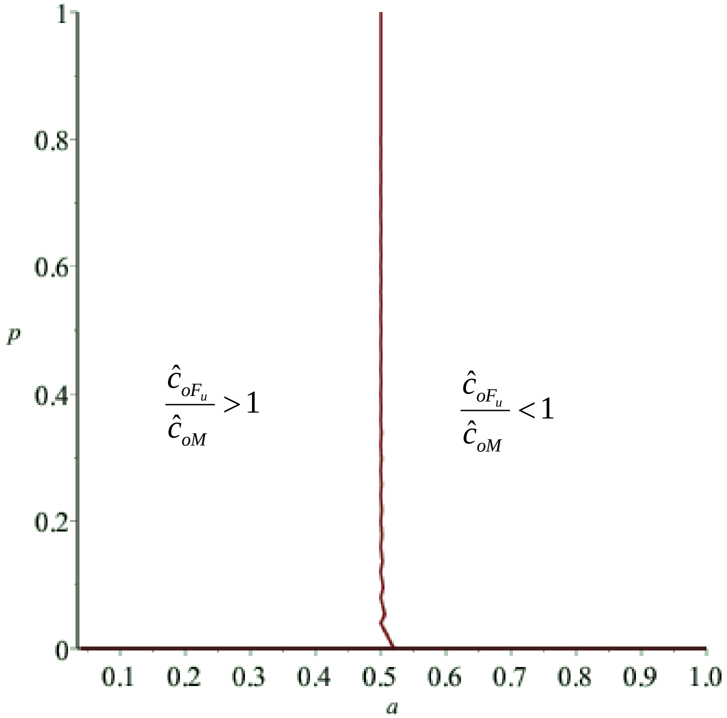
$$\frac{\hat{c}_{o,F_u}}{\hat{c}_{o,M}} = \frac{\beta(1-\alpha)A(\frac{A\alpha\beta(1-\alpha)}{\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p})^{\frac{2\alpha-1}{1-\alpha}}}{(1+\beta)(\frac{A\beta(1-\alpha)}{(1+n)(1+\beta)})^{\frac{\alpha}{1-\alpha}}(1+n)} = (\frac{\alpha(1+n)(1+\beta)}{\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p})^{\frac{2\alpha-1}{1-\alpha}}.$$

Since $\alpha(1+n)(1+\beta)-(\beta\alpha n+\beta\alpha p+\beta\alpha+\alpha n+\alpha+p)=-p(\beta\alpha+1)<0$, we have

$$\frac{\hat{c}_{o,F_u}}{\hat{c}_{o,M}} \begin{cases} >1 & if \quad 0<\alpha<0.5 \\ =1 & if \quad \alpha=0.5 \\ <1 & if \quad 0.5<\alpha<1 \end{cases}.$$

Figure 5 shows the relationship between consumption under the two policies in the relevant parameter space for the elderly unhealthy workers. A notably clean divide is predominantly made at the 0.5 value of the capital intensity of production parameter. In other words, if production is not capital intense, the consumption of the unhealthy elderly will be greater under the flexible retirement policy. For capital intense production, the reverse will be true, greater consumption under mandatory retirement policy. As we have seen from the analysis of the young unhealthy workers, flexible retirement always yields less consumption during the young stage of life. However when elderly, these same individuals could consume more under the flexible retirement plan depending upon whether the effect of lower savings dominates or whether the higher interest rate obtained dominates. In other words, since in our model the elderly will consume all their remaining savings, the unhealthy workers could consume more under flexible retirement than mandatory retirement, due to the more favorable equilibrium interest rate faced when young.

Figure 5: Consumption Comparison for Elderly Unhealthy Workers (General Setup)



Proposition 5 (Consumption when Elderly):

(a) For the healthy, the comparison of consumption when elderly depends on the level of capital intensity of production (α) and the level of population health condition (p): When both α and p are high, the consumption when elderly is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When both α and p are low, the consumption when elderly is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

(b) For the unhealthy, the comparison of consumption when elderly depends on the level of capital intensity of production (α): When $\alpha > 0.5$, the elderly stage consumption is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When $\alpha < 0.5$, the elderly stage consumption is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When $\alpha = 0.5$, the elderly stage consumption is the same under both policies.

Social aggregate level

We next compare the total social consumption in a living cohort between the two retirement policies. Such comparison may be useful from a policy perspective, if for example, policymakers would like to know the overall current welfare of current living society members depending on the two retirement policies.

Note that the social total consumption (young and elderly) for period t is given by $\hat{C}_{F,t} = pN_{-1}(1+n)^{t-1}\hat{c}_{o,F_h} + (1-p)N_{-1}(1+n)^{t-1}\hat{c}_{o,F_u} + pN_{-1}(1+n)^t\hat{c}_{y,F_h} + (1-p)N_{-1}(1+n)^t\hat{c}_{y,F_u}$ under the flexible retirement policy and $\hat{C}_{M,t} = N_{-1}(1+n)^{t-1}\hat{c}_{o,M} + N_{-1}(1+n)^t\hat{c}_{y,M}$ under the mandatory retirement policy.

Benchmark Scenario ($n = 0, \beta = 1$):

When $n = 0, \beta = 1$, we obtain the ratio of total consumption as

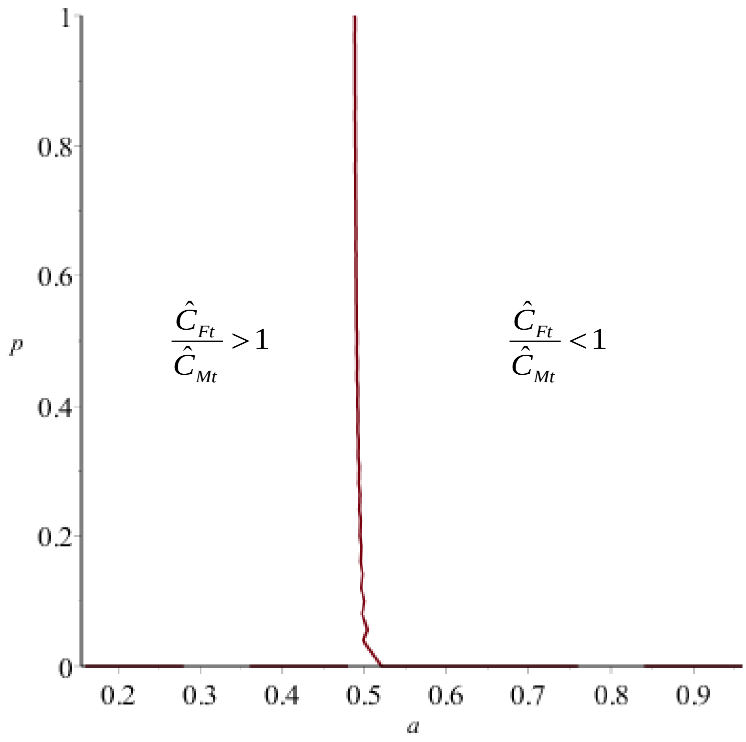
$$\frac{\hat{C}_{F,t}}{\hat{C}_{M,t}} = \frac{p\hat{c}_{o,F_h} + (1-p)\hat{c}_{o,F_u} + p\hat{c}_{y,F_h} + (1-p)\hat{c}_{y,F_u}}{\hat{c}_{o,M} + \hat{c}_{y,M}} = \frac{2(\frac{A\alpha(1-\alpha)}{\alpha p + 2\alpha + p})^{\frac{\alpha}{1-\alpha}}(p+1)(\alpha+p)}{((p+2)\alpha + p)(\frac{1}{2}A(1-\alpha))^{\frac{\alpha}{1-\alpha}}}.$$

$$= \frac{2(2\alpha)^{\frac{\alpha}{1-\alpha}}(p+1)(\alpha+p)}{((p+2)\alpha+p)^{\frac{1}{1-\alpha}}}$$

Notice that when $\alpha = 0.5$, we have $\frac{\hat{C}_{F,t}}{\hat{C}_{M,t}} = \frac{(1+p)(1+2p)}{(1+1.5p)^2} = \frac{1+3p+2p^2}{1+3p+2.25p^2} < 1$. It is also easy to verify that when $\alpha \le 0.4$, we have $\frac{\hat{C}_{F,t}}{\hat{C}_{M,t}} > 1$.

The cutoff value of 1 is plotted for this ratio as a function of the level of capital intensity of production (α) and the level of population health condition (p) in Figure 6. A similar pattern emerges compared to our previous analyses. That is, when aggregating across generations for a single time cohort, a more capital-intense production technology favors the mandatory retirement policy over the flexible retirement policy, in terms of consumption obtained.

Figure 6: Ratio of Total Consumption for Cohort *t* (Benchmark Scenario)



Calibration Scenarios:

Note, $\frac{\hat{C}_{F,t}}{\hat{C}_{M,t}} = \frac{(\alpha(1+n)(1+\beta))^{\frac{\alpha}{1-\alpha}}((1+n)\alpha+p)(((n+p+1)\beta+n)\alpha+p+1)(1+\beta)}{(((n+p+1)\beta+n+1)\alpha+p)^{\frac{1}{1-\alpha}}(1+((1+n)\beta+n)\alpha)}$. We can show the following result holds where the population growth rate is very low and the capital intensity of production is 0.5.

Lemma 1 (Social Consumption): When $n \to 0$ and $\alpha = 0.5$, the social consumption is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy. [9]

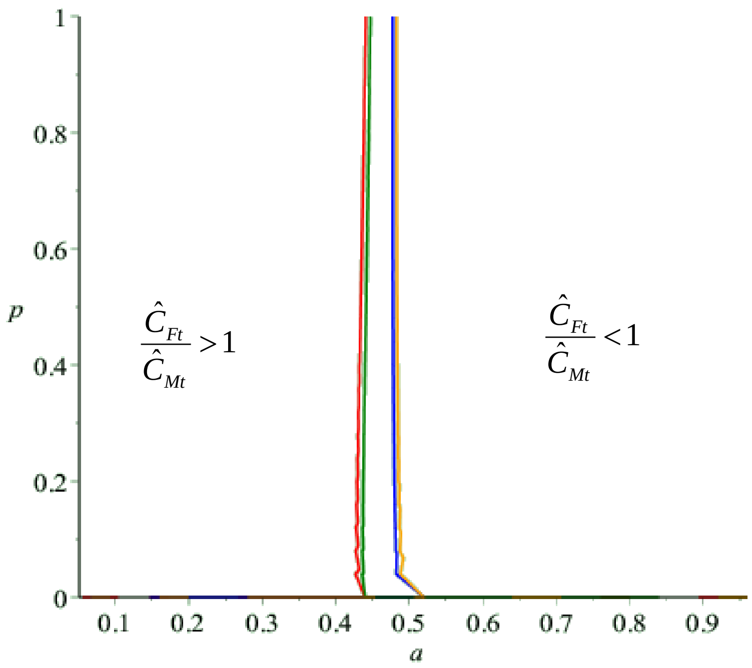
By conducting similar comparisons as in the Benchmark Scenario, we can show the cutoff curves for the social consumption comparison between the two policies in Figure 6', under each of the 4

[9] The proof is provided in the Appendix.

Calibration Scenarios, respectively.

The comparison between Figures 6 and 6' shows that the cutoff curves under different parameter values of n and β again display nearly identical qualitative patterns. A change in n seems to have little impact on the cutoff condition for the aggregate consumption comparison while an increase in β tends to shift the cutoff curve to the right (but not exceeding the vertical line where $\alpha = 0.5$).

Figure 6': Ratio of Total Consumption for Cohort *t* (Calibration Scenarios)



We summarize the comparison result for social consumption in Proposition 6.

Proposition 6 (Social Consumption): The comparison of social consumption mainly depends on the level of capital intensity of production (α): When α is high, the social consumption is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When α is low, the social consumption is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

The intuition for this result can be obtained by aggregating our previous results. The social consumption is obtained by adding up the consumption of the young individuals in society (healthy and unhealthy) as well as the elderly individuals in society (healthy and unhealthy). Since the relationship between capital intensity of production was in each case favoring the mandatory retirement policy over the flexible one for high capital intensity values, the result largely remains for aggregated social consumption.

3.2.3 Utility and Welfare

While comparison of income and consumption can capture most of the welfare implications of interest, a further step is to consider the welfare comparisons in terms of utility. There are two main considerations in the utility comparisons which differ from the income and consumption comparisons. First, individuals may have a discount factor applied to consumption in the later stage of life, following the assumption in much of economics on time discounting. Also importantly, individuals may have diminishing marginal utility of consumption via concave utility functions, which could lessen some of the welfare losses compared to the case of evaluating consumption only (equivalent to linear utility without a discount factor) if consumption is relatively high, and could heighten the differences when consumption levels are low.

Type-specific individual lifetime level

We begin with the lifetime welfare comparison between policies for healthy and unhealthy individuals respectively.

Recall from Section 2 that the healthy individual's lifetime utility function under the flexible retirement policy is given by:

$$\hat{U}_{F_h} = \ln(c_{y,F_h}) + \beta \ln(c_{o,F_h}) .$$

The unhealthy individual's lifetime utility under the flexible retirement policy is:

$$\hat{U}_{F_u} = \ln(c_{y,F_u}) + \beta \ln(c_{o,F_u}) .$$

If implementing the mandatory retirement policy, the lifetime utilities for healthy and unhealthy individuals are identically expressed as:

$$\hat{U}_M = \ln(c_{y,M}) + \beta \ln(c_{o,M}) .$$

Healthy individual

The comparison in utilities for healthy individuals under the two policies is given by the following expression:

$$\hat{U}_{F_h} - \hat{U}_M = \ln(\frac{c_{y,F_h}}{c_{y,M}}) + \beta \ln(\frac{c_{o,F_h}}{c_{o,M}}) .$$

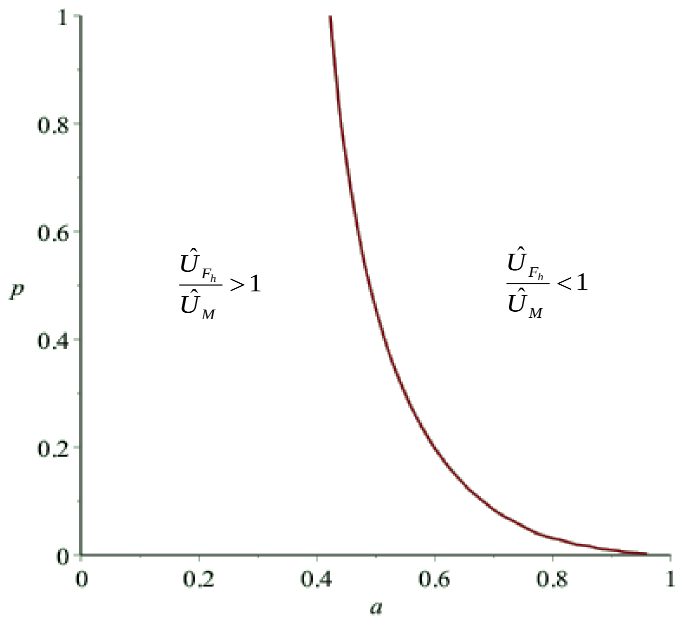
Benchmark Scenario ($n=0, \beta=1$):

Based on the results in Subsection 3.2.2, when $n=0, \beta=1$, we obtain the following expression:

$$\hat{U}_{F_h} - \hat{U}_M = \ln\left(\frac{1}{4}\frac{2^{\frac{\alpha+1}{1-\alpha}}\alpha^{\frac{3\alpha-1}{1-\alpha}}(p+1)^2(\alpha+1)^2}{((p+2)\alpha+p)^{\frac{\alpha+1}{1-\alpha}}}\right).$$

The ratio of lifetime utilities for healthy individuals is shown in Figure 7. Like most of our previous results, the welfare is lower under the flexible retirement policy for high capital intensity and high proportions of healthy individuals in the economy.

Figure 7: Ratio of Lifetime Utility of the Healthy (Benchmark Scenario)



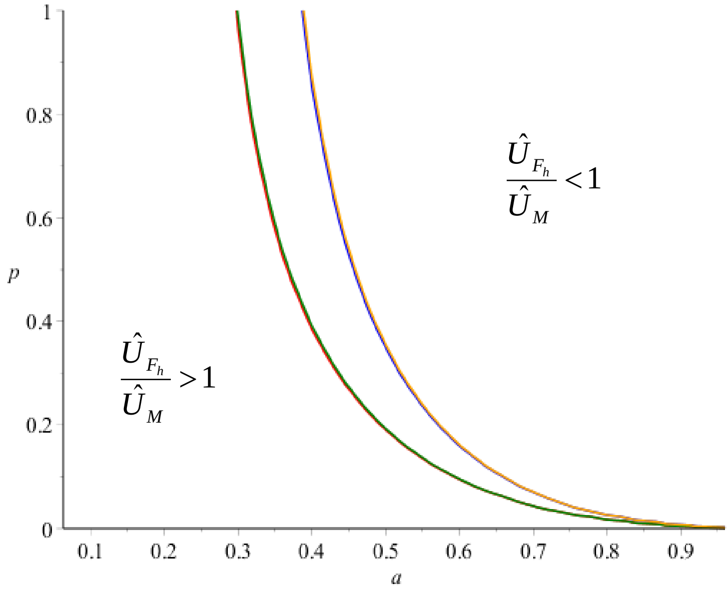
Calibration Scenarios:

Applying the results in Subsection 3.2.2, we obtain the following expression:

$$\hat{U}_{F_h} - \hat{U}_M = \ln\left(\frac{(\alpha(1+n)(1+\beta))^{\frac{\alpha+2\alpha\beta-\beta}{1-\alpha}}((\beta\alpha+1)p+\alpha(1+\beta)n+\beta+\alpha)^{1+\beta}}{((\beta\alpha+1)p+\alpha(1+n)(1+\beta))^{\frac{\beta\alpha+1}{1-\alpha}}}\right).$$

By conducting similar comparisons as in the Benchmark Scenario, we can show the cutoff curves for the healthy worker's lifetime utility comparison between the two policies in Figure 7', under each of the four Calibration Scenarios, respectively.

Figure 7': Ratio of Lifetime Utility of the Healthy (Calibration Scenarios)



The comparison between Figures 7 and 7' shows that the cutoff curves under different parameter values of n and β again have a quite robust downward-sloping pattern. Similar to many previous comparative statics analyses, a change in n seems to have little impact on the cutoff condition for the healthy worker's lifetime utility comparison while an increase in β tends to shift the cutoff curve to the right.

Unhealthy individual

The comparison in utilities for unhealthy individuals under the two policies is given by the following expression:

$$\hat{U}_{F_u} - \hat{U}_M = \ln(\frac{c_{y,F_u}}{c_{y,M}}) + \beta \ln(\frac{c_{o,F_u}}{c_{o,M}}).$$

Benchmark Scenario ($n = 0, \beta = 1$):

Based on the results in Subsection 3.2.2, when $n = 0, \beta = 1$, we obtain the following expression:

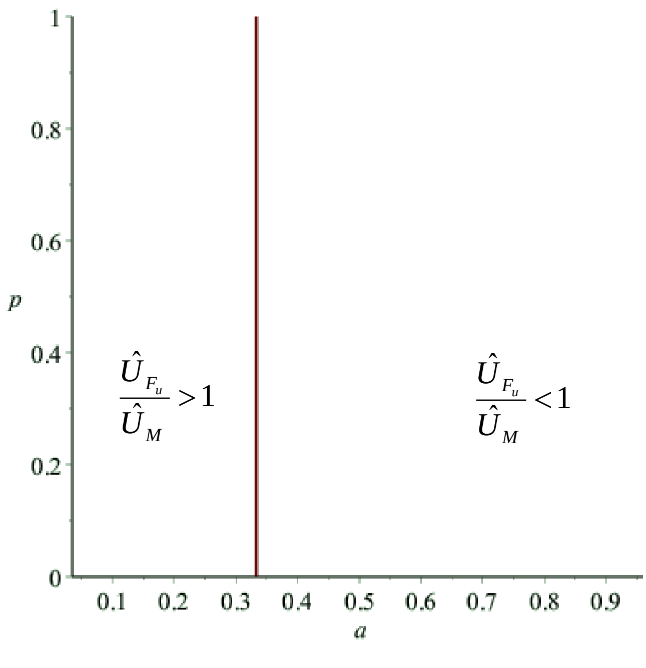
$$\hat{U}_{F_u} - \hat{U}_M = \ln((\frac{2\alpha}{(p+2)\alpha + p})^{\frac{3\alpha-1}{1-\alpha}}) = \frac{3\alpha - 1}{1-\alpha} \ln(\frac{2\alpha}{(p+2)\alpha + p}).$$

Since $2\alpha-((p+2)\alpha+p)=-p(\alpha+1)<0$, we have

$$\hat{U}_{F_u}-\hat{U}_M\begin{cases}>0 & if \quad 0<\alpha<\frac{1}{3}\\ =0 & if \quad \alpha=\frac{1}{3}\\ <0 & if \quad \frac{1}{3}<\alpha<1\end{cases}.$$

Figure 8 shows the relationship between lifetime utility under the two policies in the relevant parameter space for the unhealthy workers when $n=0,\beta=1$. A notably clean divide is predominantly made at the $\frac{1}{3}$ value of the capital intensity of production parameter. In other words, if production is not capital intense ($0<\alpha<\frac{1}{3}$), the lifetime utility of the unhealthy individual will be greater under the flexible retirement policy. For capital intense production ($\frac{1}{3}<\alpha<1$), the reverse will be true, greater consumption under mandatory retirement policy.

Figure 8: Ratio of Lifetime Utility of the Unhealthy (Benchmark Scenario)



Calibration Scenarios:

Applying the results in Subsection 3.2.2, we obtain the following expression:

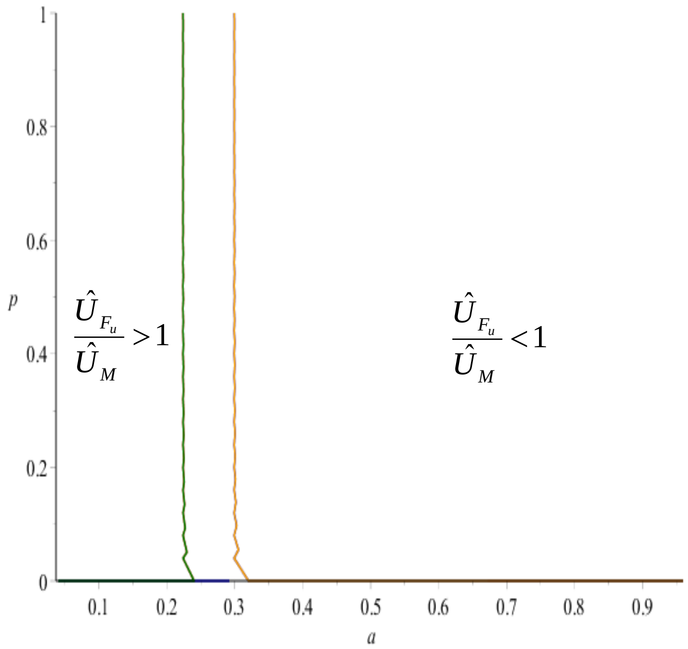
$$\hat{U}_{F_u} - \hat{U}_M = \ln\left(\frac{\alpha(1+n)(1+\beta)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p}\right)^{\frac{(2\beta+1)\alpha-\beta}{1-\alpha}} = \frac{(2\beta+1)\alpha-\beta}{1-\alpha}\ln\left(\frac{\alpha(1+n)(1+\beta)}{\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p}\right)$$

Since $\alpha(1+n)(1+\beta) - (\beta\alpha n + \beta\alpha p + \beta\alpha + \alpha n + \alpha + p) = -p(\beta\alpha + 1) < 0$, we have

$$\hat{U}_{F_u} - \hat{U}_M \begin{cases} > 0 \quad if \quad 0 < \alpha < \dfrac{\beta}{2\beta+1} \\ = 0 \quad if \quad \alpha = \dfrac{\beta}{2\beta+1} \\ < 0 \quad if \quad \dfrac{\beta}{2\beta+1} < \alpha < 1 \end{cases}.$$ [10]

Figure 8' shows the relationship between lifetime utility under the two policies in the relevant parameter space for the unhealthy workers under each of the four Calibration Scenarios. Based on the above analysis, the cutoff condition for the comparison result depends only on the value of α and β. When $\beta = 0.4010$, the cutoff condition is $\alpha \approx 0.2225$; When $\beta = 0.7397$, the cutoff condition is $\alpha \approx 0.2983$.

Figure 8': Ratio of Lifetime Utility of the Unhealthy (Calibration Scenarios)



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[10] Note that when $\beta = 1$ as, we have $\frac{\beta}{2\beta+1} = \frac{1}{3}$, which is exactly the cutoff value in the Benchmark Scenario.

We summarize the lifetime utility comparison results for both healthy and unhealthy individuals in the following proposition.

Proposition 7 (Lifetime Utility):

(a) For the healthy, the comparison of lifetime utility depends on the level of capital intensity of production (α) and the level of population health condition (p): When both α and p are high, the lifetime utility is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When both α and p are low, the lifetime utility is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

(b) For the unhealthy, the comparison of lifetime utility depends on the level of capital intensity of production (α): When $\alpha > \frac{\beta}{2\beta+1}$, the lifetime utility is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When $\alpha < \frac{\beta}{2\beta+1}$, the lifetime utility is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When $\alpha = \frac{\beta}{2\beta+1}$, the lifetime utility is the same under both policies.

As can be expected, the intuition for this proposition is much the same as the intuition for the results for consumption when young and consumption when old. The main difference between Proposition 7 and our earlier results is captured in the exact value and curvature of the relationship between technology and health parameters for healthy individuals, although the qualitative relationship is similar. For unhealthy individuals, the main difference is in the cutoff value for the technology parameter, which is lower when considering utility measures of welfare with the case where we merely consider consumption-based measures. In other words, when considering concave utility and individual discounting of utility when elderly, unhealthy individuals are less tolerant towards the flexible retirement policy. This is intuitive when considering that in our model, labor supply is unitary and unhealthy individuals are substantially poorer from a combination of lower wages and not being able to work when elderly under the flexible retirement plan.

Social aggregate level

Finally, we combine the results obtained in the previous section to consider the utility-based total welfare comparison across all individuals in the society, healthy and unhealthy. Here, the main consideration compared to the separate examination of healthy and healthy individuals' welfare is the health composition parameter p. A higher p will drive the welfare results in a direction more towards the healthy individual, whereas a lower p will drive the results towards the unhealthy individual's result.

At period t, we first calculate the social welfare under each of the two retirement policies.

For the flexible retirement policy we have

$$\hat{W}_{F,t} = pN_{-1}(1+n)^{t-1}\hat{U}_{o,F_h} + (1-p)N_{-1}(1+n)^{t-1}\hat{U}_{o,F_u} + pN_{-1}(1+n)^{t}\hat{U}_{y,F_h} + (1-p)N_{-1}(1+n)^{t}\hat{U}_{y,F_u};$$

For the case of mandatory retirement we have:

$$\hat{W}_{rt} = N_{-1}(1+n)^{t-1}\hat{U}_{or} + N_{-1}(1+n)^{t}\hat{U}_{yr}.$$

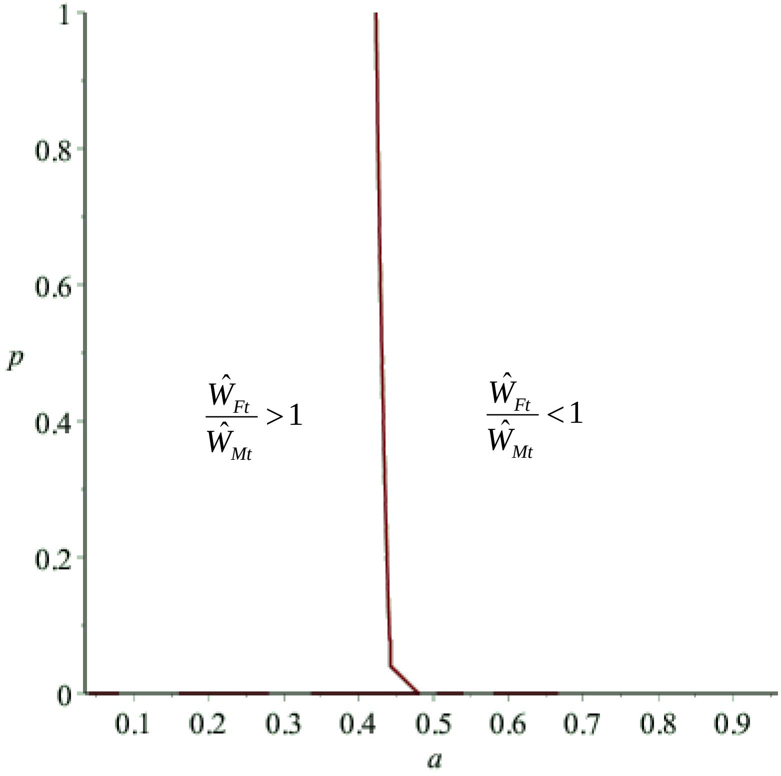
Benchmark Scenario ($n=0, \beta=1$):

When $n=0, \beta=1$, from the related calculation results of Subsection 3.2.2, we obtain

$$\frac{\hat{W}_{F,t} - \hat{W}_{M,t}}{N_{-1}} = \ln\left(\frac{(c_{o,F_h}c_{y,F_h})^{p}(c_{o,F_u}c_{y,F_u})^{1-p}}{c_{o,M}c_{y,M}}\right) = \ln\left(\frac{1}{4}\frac{\alpha^{\frac{3\alpha-1}{1-\alpha}}(1+p)^{2p}(\alpha+1)^{2p}2^{\frac{\alpha+1}{1-\alpha}}}{((p+2)\alpha+p)^{\frac{(2p-3)\alpha-2p+1}{\alpha-1}}}\right).$$

Taking the expression $\frac{1}{4}\frac{\alpha^{\frac{3\alpha-1}{1-\alpha}}(1+p)^{2p}(\alpha+1)^{2p}2^{\frac{\alpha+1}{1-\alpha}}}{((p+2)\alpha+p)^{\frac{(2p-3)\alpha-2p+1}{\alpha-1}}}$ and setting it equal to 1, we can graph the ratio of total social welfare as in Figure 9. We can observe that the result bears close similarity to our previous results in terms of the general shape of the cutoff line. One way to understand the shape of the graph is as a weighted average of Figure 7 for healthy individuals and Figure 8 for unhealthy individuals with the population proportion for these two types of individuals as the weights, which tends to yield a more linear aggregate cutoff function.

Figure 9: Ratio of Total Social Welfare at t period (Benchmark Scenario)



Calibration Scenarios:

Applying the results in Subsection 3.2.2, we obtain the following expression:

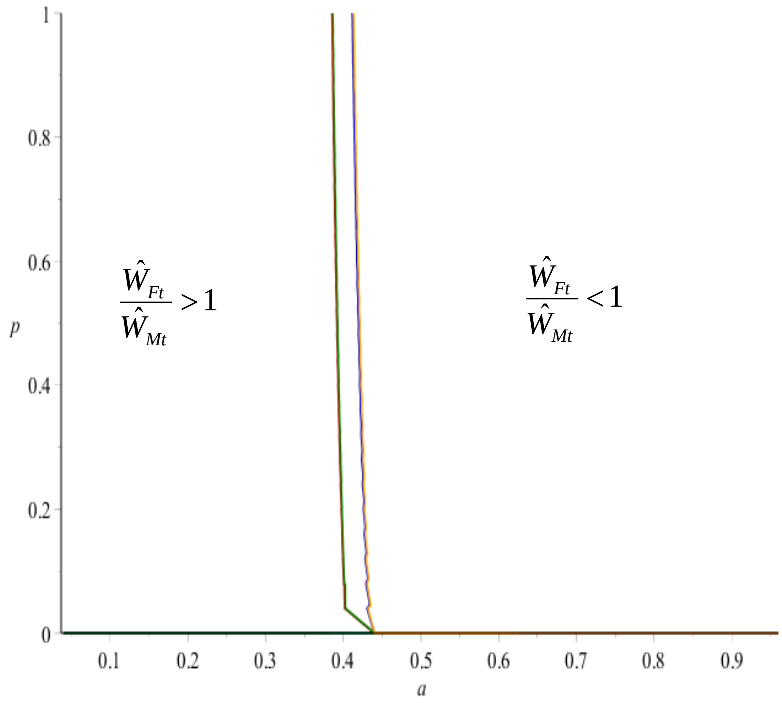
$$\frac{\hat{W}_{F,t}-\hat{W}_{M,t}}{N_{-1}}=$$

$$\ln\left(\frac{(1+\beta)\left(\frac{(((n+p)\beta+n+1)\alpha+\beta+p)^2\left(\frac{\alpha\beta(1-\alpha)}{((n+p+1)\beta+n+1)\alpha+p}\right)^{\frac{2\alpha}{1-\alpha}}(1-\alpha)}{(1+\beta)^2(((n+p+1)\beta+n+1)\alpha+p)}\right)^{p}}{\left(\frac{\beta(1-\alpha)}{(1+n)(1+\beta)}\right)^{\frac{2\alpha}{1-\alpha}}(1-\alpha)\alpha(1+n)}\cdot\left(\frac{\alpha\beta(1-\alpha)^2\left(\frac{\alpha\beta(1-\alpha)}{((n+p+1)\beta+n+1)\alpha+p}\right)^{\frac{3\alpha-1}{1-\alpha}}}{(1+\beta)^2}\right)^{1-p}\right)$$

By conducting similar comparisons as in the Benchmark Scenario, we can show the cutoff curves for the social welfare comparison between the two policies in Figure 9', under each of the four Calibration Scenarios, respectively.

The comparison between Figures 9 and 9' shows that the cutoff curves under different parameter values of n and β again display a robust pattern. Similar to many previous comparative statics analyses, a change in n seems to have little impact on the cutoff condition for social welfare comparison while an increase in β tends to shift the cutoff curve to the right.

Figure 9': Ratio of Total Social Welfare at t period (Calibration Scenarios)



We summarize the social welfare comparison result in the following Proposition.

Proposition 8 (Social Welfare): The comparison of social welfare depends on the level of capital intensity of production (α) and the level of population health condition (p): When both α and p are high, the social welfare is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy; When both α and p are low, the social welfare is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

4. Conclusion

In trying to adjust retirement policies to accommodate demographic trends, governments face a challenge in assessing the relative merits of different possible policies. For China, a country which currently has a mandatory retirement scheme with a largely uniform age of retirement imposed upon the population, one frequently discussed option is a flexible retirement scheme which allows different retirement ages for different individuals.

We formulate and analyze a general equilibrium overlapping generations model to obtain baseline assessments of these two retirement policies for a population which has heterogeneous health status and therefore heterogeneous ability to work past the standard retirement age. A main message of our analysis is the reliance of welfare comparisons on the technological and health parameters of the economy. In particular, while flexible retirement policies can indeed improve welfare under conditions of labor intensive technology and lower health status in the population, the mandatory retirement policy yields better welfare measures when the economy is capital intensive and high in health status.

Although our results are largely derived under a very basic model set up that is generalizable to real-world parameters, these findings point to a potential source of caution for societies considering a flexible retirement scheme. The main consideration is that allowing flexibility in the retirement age weakens workers' incentives to save, thus dampening the capital accumulation in the economy.[11] Thus, when the economy is highly capital intensive in its production technology and a high fraction of workers can continue to work when elderly, overall welfare tends to be diminished.

Our analysis serves as a benchmark in assessing the real-world welfare comparisons of the mandatory and flexible retirement policies. In order to quantify the actual welfare comparison as it applies to specific countries' economies, the model can be calibrated and simulated to actual parameters which are derived from economic data. A basic calibration analysis using reasonable values of China's population growth rate and time discount factors, shows that our main results are qualitatively quite robust across realistic values and variations in these parameters.

Our model has not yet incorporated several important issues pertaining to the retirement problem and actual implementation of the possible retirement schemes. We mention these issues here as directions for potential future research.

Firstly, our study has not addressed any potential pension or social security program of the government which could assist citizens in the challenge of their retirement planning (see for example, Crawford and Lilien, 1981). From this perspective, our analysis is one of retirement autarky in which the government only enforces the retirement policy but does not implement any social insurance program. Incorporating social insurance into the analysis is a key feature to add realistic policy relevance, and our current results only provide a benchmark case without such additional government transfer programs.

Another potential direction for our model set up is to endogenize the labor market decision of elderly workers. In the present model, although older workers are heterogeneous in their health status,

[11] In this aspect, our study adds to the literature on determinants of consumption in the Chinese economy (Lien, Peng and Zheng, 2017; Lian, He, Li, Lien, and Zheng, 2018).

it is assumed that healthy workers will work and unhealthy workers will retire. This raises a potential moral hazard issue since workers' health statuses are imperfectly observable to governments. A reasonable next step in enriching the model may be to allow workers to make their own decisions about employment based on their private health information and other influencing factors.

We finally note that our model uses a representative agent approach in which the only source of heterogeneity is workers' health status when elderly. However, as suggested by several studies mentioned in the introduction, another possible direction to generate an even richer set of policy simulations and predictions is to allow for correlation between health status and some of the key demographic variables in society, such as gender, occupation, income or geographic region.

As world populations continue to exhibit an aging trend and governments consider possible policy interventions on individuals' retirement decisions, the analysis of retirement policies remains an important topic. While flexible retirement schemes often hold strong popular appeal as in the case of China, an important consideration for economists, policymakers and citizens is the general equilibrium consequence of policies, which are not always immediately apparent from the stated policy alone. Theoretical modeling, policy simulation with realistic parameters, and assessing the appeal of different policies to subsets of the population are all essential ingredients to understanding the ideal retirement policy for a society.

References:

Anderson, K. H., & Burkhauser, R. V. (1985). The retirement-health nexus: A new measure of an old puzzle. *The Journal of Human Resources*, 20(3), 315-330.

András, S. (2003). Designing optimal linear rules for flexible retirement. *Journal of Pension Economics and Finance*, 2(3), 273-293.

Arrow, K. (1995). Intergenerational equity and the rate of discount in long-term social investment. Paper presented at the IEA World Congress, December, Tunis, Tunisia.

Banister, J., Bloom, D. E., & Rosenberg, L. (2012). Population aging and economic growth in China. *Palgrave Macmillan UK*, 93(5), 711-718.

Bazzoli, G. J. (1985). The early retirement decision: New empirical evidence on the influence of health. *Journal of Human Resources*, 20(2), 214-234.

Cohen, A., & Siegelman, P., Testing for Adverse Selection in Insurance Markets. *Journal of Risk and Insurance*, 77(1), 39 – 84.

Crawford, V. P., & Lilien, D. M. (1981). Social security and the retirement decision. *Quarterly Journal of Economics*, 96(3), 505 - 529.

David, W. J., & Lee, W. S., (2009). Retiring to the good life? The short-term effects of retirement on health. *Economics Letters*, 103(1), 8-11.

Dwyer, D. S., & Mitchell, O. S. (1999). Health problems as determinants of retirement: Are self-rated measures endogenous? *Journal of Health Economics*, 18(2), 173–193.

Ekerdt, D. J. , Bosse, R., & LoCastro, J. S. (1983). Claims that retirement improves health. *Journal of Gerontology*, 38(2), 231-236.

Elena, S. H., Yin, Y., Kean, M., Slymen, D. J., Liu, W. T., Zhang, M. Y., & Katzman, R. (1998). Self-perceived health and 5-year mortality risks among the elderly in Shanghai, China. *American Journal of Epidemiology*, 147(9), 880-90.

Eső, P., & Simonovits, A. (2002). Designing optimal benefit rules for flexible retirement. Discussion Paper CMS-EMS 1353, Northwestern University, Evanston, IL.

Evans, D. (2006). Social discount rates for the European Union. Working Paper No. 2006–20, Fifth Milan European Economy Workshop, Universita degli Studi di Milano, Italy.

Feng, Z. H., Lien, J. W., & Zheng, J. (2017). Welfare implications of a flexible retirement policy. Working Paper.

Feng, Z., Wang, W. W., & Jones, K. (2013). A multilevel analysis of the role of the family and the state in self-rated health of elderly Chinese. Health & Place, 23(23C), 148-156.

Giles, J., & Mu, R. (2007). Elderly parent health and the migration decisions of adult children: evidence from rural China. *Demography*, 44(2), 265-288.

Gu, D., Dupre, M. E., Warner, D. F., & Zeng, Y. (2009). Changing health status and health expectancies among older adults in China: Gender differences from 1992 to 2002. *Social Science & Medicine*, 68(12), 2170–2179.

Gunderson, M. (1998). Flexible retirement as an alternative to 65 and out. Book, Series: Commentary; no. 106, Publisher: [Toronto] : C.D. Howe Institute, ISBN: 0888064349.

Hesketh, T., Ye, X. J., Li, L., & Wang, H. M. (2008). Health status and access to health care of migrant workers in China. *Public Health Reports*, 123(2), 189-197.

Impavido, G., Hu, Y. W., & Li, X. (2009). Governance and fund management in the Chinese pension system. *Social Science Electronic Publishing*, 09(246), 1-50.

James, E. (2003). How can China solve ist Old Age Security Problem? The Interaction Between Pension, SOE and Financial Market Reform. *Comparative Economic & Social Systems*, 1(1), 53-75.

John, S., Lei, X. Y., Park, A., Shen, Y., Smith, J. P., Yang, Z., & Zhao, Y. H. (2010). Health outcomes and socio-economic status among the elderly in China: evidence from the CHARLS pilot. *Journal of Population Agein*, 3(3-4), 111-142.

Li, H., & Zhou, L. A. (2005). Political turnover and economic performance: the incentive role of personnel control in China. *Journal of Public Economics*, 89(9–10), 1743-1762.

Lian, Z., He, R., Li, H.Q., Lien, J. W., & Zheng, J. (2018) Political style, individual preference and consumption pattern: A behavioral approach. Working Paper.

Lazear, E. P. (1979). Why is there mandatory retirement? *Journal of Political Economy*, 87(6), 1261-1284.

Lien, J., Peng, Q. Q., & Zheng, J. (2017) Major earthquakes and present-focused expenditures. Working Paper.

Lien, J., Wang, W., & Zheng, J. (2016). A model of capital allocation, education and job choice in China, *The Chinese Economy, Special Issue: Some Challenges to Economic Growth and Stability in China*, 49(5), 307-326.

Liu, T., & Liu, J. (2017). Two-child policy, gender income and fertility choice in China. MPRA Paper.

Liu, X., Liang, J., & Gu, S. (1995). Flows of social support and health status among older persons in China. *Social Science & Medicine*, 41(8), 1175-84.

Martin, L. G. (1988). The Aging of Asia. *Journal of Gerontology: SOCIAL SCIENCES*, 43(4), S99-113.

McGarry, K. (2004). Health and retirement: Do changes in health affect retirement expectations? *The Journal of Human Resources*, 39(3), 624-648.

National Bureau of Statistics of China (2018). China Statistical Abstract 2018, ISBN 978-7-5037-8461-3, Beijing.

Nordhaus, W. (1993). Rolling the 'DICE': An optimal transition path for controlling greenhouse gases. *Resource and Energy Economics*, 15, 27–50.

Palmore, E. (1972). Compulsory versus flexible retirement: Issues and facts. *Gerontologist*, 12(4), 343-348.

Peng, Y. I., Liu, T. C., Chen, C. S., & Chan, Y. S. (2016). Does poor health raise preferences for retirement? *The Social Science Journal*, 53(4), 4501-509.

Quinn, J. F. (1977). Microeconomic determinants of early retirement: A cross-sectional view of white married men. *The Journal of Human Resources*, 12(3), 329-346.

Smith, J. P. (1999). Healthy bodies and thick wallets: the dual relation between health and economic status. *Journal of Economic Perspectives*, 13(2), 145-166.

Song, S., Chu, S.F. (1997). Social security reform in china: the case of old-age insurance. *Contemporary Economic Policy*, 15(2), 85-93.

Sun, R., & Gu, D. (2008). Air pollution, economic development of communities, and health status among the elderly in urban China. *American Journal of Epidemiology*, 168(11), 1311-1318.

Tang, K. L., & Ngan, R. (2010). China: developmentalism and social security. *International Journal of Social Welfare*, 10(4), 253-259.

Vuuren, D. V. (2014). Flexible retirement. *Journal of Economic Surveys*, 28(3), 573–593.

Wang, F., Zhu, J., Yao, P., Li, X. L., He, M. A., Liu, Y. W., Yuan, J., Chen, W. H., Zhou, L., Min, X. W., Fang, W. M., Liang, Y., Wang, Y. J., Wei, S., Liu, J. C., Miao, X. P., Lang, M. J., Jiang, X. X., Zhang, P., Li, D. F., Lu, C. W., Wang, X. Z., Shi, W. H., Zheng, J. R., Guo, H., Zhang, X. M., Yang, H. D., Hu, F. B., & Wu, T. C. (2013). Cohort Profile: the Dongfeng-Tongji cohort study of retired workers. *International Journal of Epidemiology*, 42(3), 731-740.

Wang, W., Li, Q., & Lien, D. (2016). Human capital, political capital, and off-farm occupational choices in rural China, *International Review of Economics and Finance*, 42, 412-422.

Yu, M. Y., & Sarri, R. (1997). Women's health status and gender inequality in China. *Social Science & Medicine*, 45(12), 1885-1898.

Zhang, L. & Wang, H. (2008). Dynamic process of adverse selection: Evidence from a subsidized community-based health insurance in rural China. *Social Science & Medicine*. 67(7), 1173-1182.

Appendix of Proofs:

Corollary 1 (Wage): The equilibrium wage is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Proof: By Proposition 1 we have $\hat{k}_F < \hat{k}_M$. Thus we have $A(1-\alpha)(\hat{k}_F)^{\alpha} < A(1-\alpha)(\hat{k}_M)^{\alpha}$. Since in equilibrium we know $\hat{w} = A(1-\alpha)\hat{k}^{\alpha}$, therefore we have $\hat{w}_F < \hat{w}_M$.

Corollary 2 (Interest Rate): The equilibrium interest rate is higher under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Proof: By Proposition 1 we have $\hat{k}_F < \hat{k}_M$. Thus we have $\alpha A(\hat{k}_F)^{\alpha-1} > \alpha A(\hat{k}_M)^{\alpha-1}$. Since in equilibrium we know $\alpha A\hat{k}^{\alpha-1} = \hat{R}$, therefore we have $\hat{R}_F > \hat{R}_M$.

Lemma 1 (Social Consumption): When $n \to 0$ and $\alpha = 0.5$, the social consumption is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.

Proof: Note that when $\alpha = 0.5$,

$$\frac{\hat{C}_{F,t}}{\hat{C}_{M,t}} = \frac{(0.5(1+n)(1+\beta))(0.5(1+n)+p)(0.5((n+p+1)\beta+n)+p+1)(1+\beta)}{(0.5((n+p+1)\beta+n+1)+p)^2(1+0.5((1+n)\beta+n))}$$

$$= \frac{0.5(1+n)(1+\beta)^2[p+0.5(1+n)][(1+0.5\beta)p+0.5\beta(1+n)+1+0.5n]}{[0.5\beta(1+n)+1+0.5n][(1+0.5\beta)p+0.5(1+\beta)(1+n)]^2}$$

$$= \frac{0.5(1+n)(1+\beta)^2\{(1+0.5\beta)p^2+[0.75\beta(1+n)+1.5+n]p+0.5(1+n)(0.5\beta(1+n)+1+0.5n)\}}{[0.5\beta(1+n)+1+0.5n][(1+0.5\beta)^2p^2+(1+0.5\beta)(1+\beta)(1+n)p+0.25(1+\beta)^2(1+n)^2]}$$

$$= \frac{0.5(1+n)(1+\beta)^2(1+0.5\beta)p^2+0.5(1+n)(1+\beta)^2[0.75\beta(1+n)+1.5+n]p+0.25(1+n)^2(1+\beta)^2[0.5\beta(1+n)+1+0.5n]}{[0.5\beta(1+n)+1+0.5n](1+0.5\beta)^2p^2+[0.5\beta(1+n)+1+0.5n](1+0.5\beta)(1+\beta)(1+n)p+0.25[0.5\beta(1+n)+1+0.5n](1+\beta)^2(1+n)^2}$$

In the expression above, first notice the third term in the numerator is the same as the third term in the denominator.

Then, we compare the coefficient of the first term in the numerator with the coefficient of the first term in the denominator. As $n \to 0$,

$$0.5(1+n)(1+\beta)^2(1+0.5\beta)-[0.5\beta(1+n)+1+0.5n](1+0.5\beta)^2$$

$$\approx 0.5(1+\beta)^2(1+0.5\beta)-(0.5\beta+1)(1+0.5\beta)^2=(1+0.5\beta)[0.25\beta^2-0.5]<0 \quad \text{since} \quad \beta\le 1.$$

Lastly we compare the coefficient of the second term in the numerator with the coefficient of the second term in the denominator.

$$\begin{aligned}
&0.5(1+n)(1+\beta)^2[0.75\beta(1+n)+1.5+n]-[0.5\beta(1+n)+1+0.5n](1+0.5\beta)(1+\beta)(1+n)\\
&=0.5(1+n)(1+\beta)\{(1+\beta)[0.75\beta(1+n)+1.5+n]-(2+\beta)[0.5\beta(1+n)+1+0.5n]\}\\
&=0.125(1+n)(1+\beta)\{[3(1+n)\beta^2+(7n+9)\beta+4n+6]-[2(1+n)\beta^2+(6n+8)\beta+4n+8]\}\\
&=0.125(1+n)(1+\beta)\{(1+n)\beta^2+(1+n)\beta-2\}
\end{aligned}$$

As $n\to 0$, $(1+n)\beta^2+(1+n)\beta-2\to\beta^2+\beta-2\le 0$ since $\beta\le 1$.

Given $n\to 0$, since all the terms in the numerator are less than or equal to the corresponding terms in the denominator, and the first term in the numerator is less than the first term in the denominator, we have shown that the numerator is less than the denominator. This means, when $n\to 0$ and $\alpha=0.5$, the social consumption is lower under the Flexible Retirement Policy than under the Mandatory Retirement Policy.